A NOTE ON PERMUTATIONS IN AN ARBITRARY FIELD

L. CARLITZ

The writer [1] has proved that every permutation on the numbers of the finite field $GF(q)$ is generated by the special permutations
\[(1) \quad x^{q-2}, \quad ax + \beta \quad (\alpha, \beta \in GF(q), \alpha \neq 0).\]

Let $F$ denote an arbitrary field. Define the function
\[(2) \quad x^* = \begin{cases} x^{-1} & (x \in F, x \neq 0), \\ 0 & (x = 0). \end{cases}\]

Clearly $x^*$ defines a permutation of $F$.

The following theorem holds.

**Theorem 1.** Every transposition $(\alpha \beta)$, where $\alpha, \beta \in F$ is finitely generated by the special permutations
\[(3) \quad x^*, \quad \gamma x + \delta \quad (\gamma, \delta \in F, \gamma \neq 0).\]

The proof (compare [1]) follows from consideration of the function
\[g(x) = -\alpha^2 \left( (x - \alpha)^* + \frac{1}{\alpha} \right)^* - \alpha,*\]
where $\alpha$ is a fixed nonzero number of $F$. It is easily verified that $g(x)$ represents the transposition $(0\alpha)$.

Let $G = G(F)$ denote the group consisting of all finite products
\[t_1 t_2 \cdots t_n,\]
where the $t_j$ are arbitrary transpositions $(\alpha \beta)$. As an immediate corollary of Theorem 1 we have

**Theorem 2.** The group $G$ is generated by the special permutations (3).

**Reference**


Duke University

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