Theorem. If $\sum_{n=1}^{\infty} b_n = f$ is a Fourier-Stieltjes-series, then $\sum_{n=1}^{\infty} B_n \sin nt \in L$ and $\sum_{n=1}^{\infty} B_n \cos nt \in L$, where $B_n = n^{-1} \sum_{j=1}^{n} b_j$. Or in symbols: $\mathcal{T}_H \in (dV, L)$.

Sketch of proof. $\mathcal{T}_H$ is a linear bounded transformation from $L_*$ into $L_*$ [10, p. 471] and $\mathcal{T}_H \in (L_*, L_*)$ implies $\sup_n \|T_n\| < \infty$ [2, Theorem 4.4] where $T_n = \sup_{|f|_{L_2} \leq 1} \|T_n f\|_L$ and $T_n f = \sum_{j=1}^{n} (1 - j/(n+1))B_j \sin j$. Since $dV$ is a norm determining manifold in $L$ and since $\|f\|_L = \|f\|_{dV}$ for $f \in L$ we have also $\|T_n\| = \sup_{|f|_{dV} \leq 1} \|T_n f\|_{dV} = O(1) \ (n \to \infty)$.

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and therefore $T_H \in (dV_1, dV_2)$ [2, Theorem 4.5]. (Correspondingly we get $T_H \in (dV_1, dV_3)$ but this is of no interest here.)

By Kinukawa and Igari [6, p. 274] we have $T_H \in (L_1, L_2)$ and since $T_H \in (L_1, L_3)$ we have $T_H \in (L_1, L)$. The proof that $T_H \in (L_1, L)$ implies $T_H \in (dV_1, dV)$ is exactly the same as the proof that $T_H \in (L_1, L)$ implies $T_H \in (dV_1, dV_3)$. Since $dV = L$ [8; 11, p. 285] we have $T_H \in (dV_1, L)$.

Remarks. 1. Let $V$ be the space of Fourier coefficients of functions of bounded variation. From the fact that $b \in V$ implies $\sum_{n=1}^{\infty} |b_n| < \infty$ [5; 11, p. 286] it follows with our theorem that $b \in dV$ implies $\sum_{n=1}^{\infty} n^{-2} |\sum_{j=1}^{n} b_j| < \infty$.

2. As remarked by Loo [7, p. 270] we have $T_H \in (L_1, L_3)$.

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