where \( \tau = 1 - y \). Now as \( y \to 1 \), \( \tau \to 0 \), and it is known \[2\] that the last integral approaches \( f(x) \) almost everywhere as \( \tau \to 0 \). This proves (10).

**Bibliography**


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**ON HYPONORMAL OPERATORS**

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A bounded linear operator \( T \) on a Hilbert space \( \mathcal{H} \) is said to be **hypotormal** in case \( \| T^*x \| \leq \| Tx \| \) for all \( x \in \mathcal{H} \). This short note gives a negative answer to the question raised in [1, p. 188]: "Does there exist a completely continuous hypotormal operator which is not normal?"

**Theorem.** If \( T \) is hypotormal, \( \| T^n \| = \| T \|^n \) for all \( n \).

**Proof.** It is sufficient to prove that \( \| T \| = 1 \) implies \( \| T^n \| = 1 \) for all \( n \). Consider the following property:

\( (C_n) \) For every \( \epsilon > 0 \), there exists a unit vector \( x \) such that

\[ \| T^nx \| \geq 1 - \epsilon \quad \text{and} \quad \| T^nx - T^*T^{n+1}x \| \leq \epsilon. \]

\( (C_n) \) just says that 1 is an approximate proper value for the self-adjoint operator \( T^*T \) (see [1, p. 170]). \( (C_n) \) obviously implies \( \| T^* \| = 1 \). Now suppose that \( (C_n) \) is valid. For \( \epsilon > 0 \) and \( x \) (indicated in \( (C_n) \))

\[
\| T^{n+1}x - T^*T^{n+1}x \|^2 \\
= \| T^{n+1}x \|^2 - 2\| T^{n+2}x \|^2 + \| T^*T^{n+2}x \|^2 \\
\leq \| T^nx \|^2 - \| T^{n+2}x \|^2 \quad \text{(because} \quad \| T \| = \| T^* \| = 1) \\
\leq \| T^nx \|^2 - \| T^*T^{n+1}x \|^2 \quad \text{(because} \quad T \text{ is hypotormal)} \\
\leq \| T^nx - T^*T^{n+1}x \| \{ \| T^nx \| + \| T^*T^{n+1}x \| \} \leq 2\epsilon \quad \text{by} \quad (C_n).
\]

Received by the editors February 12, 1962.
Also
\[ \| T^{n+1}x \| \geq \| T^{n+2}x \| \geq \| T^* T^{n+1}x \| \]
\[ \geq \| T^n x \| - \epsilon \geq 1 - 2\epsilon \text{ by (C").} \]

Since \( \epsilon > 0 \) is arbitrary, (\( C_{n+1} \)) is valid.

**Corollary 1.** Every nonzero hyponormal operator has a nonzero element in its spectrum.

This follows from the above theorem via the known fact that the spectral radius of an operator \( T \) is equal to \( \lim_{n \to \infty} \sqrt[n]{\| T^n \|} \).

**Corollary 2.** Every completely continuous hyponormal operator is normal.

**Proof.** Let \( T \) be hyponormal and completely continuous. In view of a known property of a hyponormal operator (see [1, p. 168]) it is sufficient to prove that the set of all proper vectors for \( T \) is total, in other words, the set \( \mathcal{M} \) of all vectors orthogonal to every proper vector consists of only the null vector. Since \( \mathcal{M} \) reduces \( T \) (see [1, p. 168]), the restriction of \( T \) to \( \mathcal{M} \), denoted by \( T|\mathcal{M} \), is also hyponormal. The spectrum of \( T|\mathcal{M} \) consists of 0 only, for \( T|\mathcal{M} \) is completely continuous and has no proper value by the definition of \( \mathcal{M} \). By the above corollary this means that \( T|\mathcal{M} = 0 \) or \( \mathcal{M} = \{0\} \). The former is obviously excluded.

**Reference**


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