

## REFERENCES

1. Richard Arens, *Extension of functions on fully normal spaces*, Pacific J. Math. **2** (1952), 11–22.
2. O. Hanner, *Retraction and extension of mappings of metric and non-metric spaces*, Ark. Mat. **2** (1952), 315–360.
3. F. Hausdorff, *Erweiterung einer stetigen Abbildung*, Fund. Math. **30** (1938), 40–47.
4. C. W. Saalfrank, *A generalization of the concept of absolute retract*, Proc. Amer. Math. Soc. **12** (1961), 374–378.

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## A COVERING THEOREM FOR CONVEX MAPPINGS<sup>1</sup>

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The following theorems are classical. Proofs can be found in [1, pp. 214, 223].

**THEOREM 1.** *If  $f(z)$  is regular and univalent in  $|z| < 1$ ,  $f(0) = 0$  and  $f'(0) = 1$  then the image domain covers the circle  $|w| < 1/4$ .*

**THEOREM 2.** *If  $f(z)$  is regular and univalent in  $|z| < 1$ ,  $f(0) = 0$ ,  $f'(0) = 1$  and the image  $D$  is convex, then  $D$  covers the circle  $|w| < 1/2$ .*

The purpose of this note is to show that Theorem 2 can be proven as a simple consequence of Theorem 1. Suppose then that  $f(z)$  satisfies the hypotheses of Theorem 2 and  $f(z) \neq c$ . Let  $g(z) = (f(z) - c)^2$ . Suppose that  $z_1$  and  $z_2$  are distinct points in the unit circle and  $g(z_1) = g(z_2)$ . Then either  $f(z_1) = f(z_2)$  or  $2^{-1}(f(z_1) + f(z_2)) = c$ . The first equation cannot hold since  $f(z)$  is univalent in  $|z| < 1$ . Neither can the second equation hold, for  $D$  is convex and therefore the average of every two points in  $D$  is also in  $D$ . This proves that  $g(z)$  is univalent in  $|z| < 1$ . The function  $h(z) = (c^2 - g(z))/2c$  satisfies the hypotheses of Theorem 1, and  $h(z) \neq c/2$  since  $f(z) \neq c$ . Therefore,  $|c/2| \geq 1/4$ ,  $|c| \geq 1/2$ .

## REFERENCE

1. Z. Nehari, *Conformal mapping*, New York, McGraw-Hill, 1952.

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