ON \((n-1)\)-DIMENSIONAL FACTORS OF \(I^n\)

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We recall that the cone \(C(H)\) over a space \(H\) is defined as the product \(H \times [0, 1]\) with each \((h, 0), h \in H\), identified to a point \(v\) called the vertex. The part of \(C(H)\) that corresponds to \(H \times [0, 1]\) is the open cone, denoted by \(OC(H)\), over \(H\).

**Theorem 1.** Let \(K\) denote the cone over a compact Hausdorff space \(H\). If the vertex of \(K\) has a neighborhood equivalent to \(E^n\) then \(K \times I\) is a topological cube.

As an immediate consequence, we obtain an affirmative answer to a question raised by Kwun and Raymond [7].

**Theorem 2.** For each \(n \geq 5\), there exists a space \(X\) such that \(X \times I = I^n\) and \(\text{Bd} X\) contains no \((n-2)\)-cell.

Here, equality means topological equivalence. It may be noted that Theorem 2 does not hold for \(n \leq 4\) by a theorem due to Bing [1].

**Proof of Theorem 1.** A proof that \(OC(H) = E^n\) is available in various works such as [6]. Hence by [8, Proposition 3.1] \(\text{Bd} K = H\) and \(\text{Int} K = OC(H)\) as a generalized manifold with boundary, so we must have \([11] \text{Bd}(K \times I) = (\text{Bd} K \times I) \cup (K \times \text{Bd} I)\), and \(\text{Int}(K \times I) = K \times \text{Int} I = OC(H) \times E^1 = E^{n+1}\).

First, we observe \(H \times E^1 = OC(H)\) minus vertex = \(S^{n-1} \times E^1\). By taking two point compactification, we see \(\text{Bd}(K \times I)\) = suspension over \(H = S^n\). Note that uniqueness of this compactification follows from the uniqueness of one point compactification.

Each point of \(K\) has a neighborhood homeomorphic to \(H \times I\) or \(E^n\). To prove that \(K \times I\) is a manifold with boundary we merely need to show that \(H \times I^2\) is a manifold with boundary. But, this follows from the facts: (1) \(H \times E^1 = S^{n-1} \times E^1\), and (2) each point of \(I^2\) has a neighborhood equivalent to \(E^1 \times I\).

Now, \(\text{Bd}(K \times I)\) is collared [3]. Hence \(K \times I\), being an \((n+1)\)-cell plus an \((n+1)\)-annulus around it, is a cell by the generalized Schoenflies theorem [2].

**Proof of Theorem 2.** Let \(Y\) be the one point compactification of \((n-2)\)-dimensional factor of \(E^{n-1}\), \(n \geq 5\), described by Kwun [5] or Rosen [12]. Clearly, the boundary of cone \(X\) over \(Y\) contains no \((n-2)\)-cell but suspension over \(Y\) is an \(S^{n-1}\). Hence by Theorem 1 \(X \times I = I^n\).

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REMARKS. It will not be difficult to verify that our most non-
euclidean factor $X$ is, in fact, the hyper space of an upper semi-
continuous decomposition of $I^{n-1}$ having densely distributed arcs in
$\text{Bd } I^{n-1}$ as nondegenerate elements. On the other hand, examples of
Poénaru [10], Mazur [9] and Curtis [4] show that not all factors
of cubes are conical neighborhoods. It will be interesting to know if
$X \times I = I^n$ implies that $\text{Bd } X$ is collared.

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