The space $M$ is called pseudo-isotopically contractible provided that if $A$ is a compact subset of $M$ there is a continuous function $r(x, t)$ of $M \times [0, 1]$ into $M$ such that (1) if $t < 1$, $r|_{M \times t}$ is a homeomorphism onto, and (2) if $t = 1$, $r|_{A \times 1}$ is a point.

Let $X$ be a locally euclidean $n$-dimensional space with the property that each pair of points lies in the interior of some $n$-ball. Clearly $X$ is a connected $n$-manifold without boundary.

**Theorem.** If $M$ is a locally euclidean $n$-dimensional space with the property that each pair of points lies interior to some $n$-ball, then $M = A$ is an open $n$-cell if and only if $M$ is pseudo-isotopically contractible.

**Lemma.** If $M$ is pseudo-isotopically contractible and $p$ is a point of $M$, the function $r(x, t)$ may be chosen so that $r(A, 1) = p$.

**Proof.** Let $U$ be the interior of a ball containing $p$ and $q$. Suppose $U$ is given a co-ordinate system $(x_1, \ldots, x_n)$, where $x_1^2 + \cdots + x_n^2 < 1$ and $(x_1, \ldots, x_n) \in \overline{U} \setminus U$ if and only if $x_1^2 + \cdots + x_n^2 = 1$.

Let $0 < \epsilon < 1$; then the mapping

\[ x'_i = x_i + \epsilon \rho, \quad 0 \leq t \leq 1, \]

\[ x'_i = x_i, \quad i > 1, \]

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where $\rho$ is the distance from $(x_1, \cdots, x_n)$ to $\overline{U\setminus U}$, defines an isotopy of $U$ on itself that is fixed on $\overline{U\setminus U}$ and carries the origin into $(\epsilon, 0, 0, \cdots, 0)$.

The composition of a finite number of such isotopies will give the desired result.

**Proof of the Theorem.** Since $M$ is locally separable and connected, it is separable. Since $M$ is also locally compact, $M = \bigcup_i^n A_i$, where $A_i$ is compact and $A_{i+1} \supset A_i$. Let $B_i$ denote the closure of the spherical $1/i$ neighborhood of $\rho$.

To each $i = 1, 2, \cdots$, there is a continuous function $r(x, t; i)$ on $M \times [0, 1]$ that is a homeomorphism onto for $t < 1$, and, for $t = 1$, $A_i$ is contracted to $\rho$. Let us choose $t_i < 1$ such that $r(x, t; i)$ shrinks $A_i \times t_i$ to a subset of the interior of $B_i$. Suppose $r^{-1}(x, t_i; i)$ maps $B_i$ on $E_i$. Then $E_i$ is an open $n$-cell and $M = \bigcup_i^n E_i$. For, if $q \in M$, $q \in A_j$ for some $j$. Then $r(x, t_i; j)$ carries $q$ onto $q^1$ in $B_i$ and $r^{-1}(x, t_i; j)$ carries $q^1$ back onto $q$.

By a recent result of M. Brown, $M$ must be homeomorphic to euclidean $n$-space, $E^n$ [1].

**Reference**


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