2. Let $\hat{H}^n(U, A) \cong \hat{H}^n(U, B)$ in two successive dimensions and for all subgroups but do not assume the isomorphisms induced by a module homomorphism. It would not be reasonable to expect isomorphisms for all $n$ and all subgroups. The following counterexample justifies our pessimism. Let $G = G_3(a, b: a^3 = b^7 = 1, aba^{-1} = b^2)$; let $A$ be $\mathbb{Z}$ with trivial action and $B$ the result of dimension shifting down two steps. Then $\hat{H}^q(G, A; 7) = \hat{H}^{q-2}(G, B; 7) = 0$ for $q = 1, 2, 3, 4, 5$ and $\hat{H}^4(G, A; 7) = \hat{H}^4(G, B; 7) \neq 0$.

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QUASI-INVERTIBLE PRIME IDEALS

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In this note $R$ will denote a commutative ring with unit and a proper ideal of $R$ is an ideal of $R$ different from $(0)$ and $R$. Nakano has shown that $R$ is a Dedekind domain, provided that every proper prime ideal of $R$ is invertible [1]. In [2], Krull defines a prime ideal $P$ to be quasi-invertible provided $PP^{-1} > P$, where $>$ denotes proper containment and $P^{-1}$ is the set of elements $x$ in the total quotient ring of $R$ such that $xP \subset R$. The purpose of this note is to prove that Nakano's result remains valid when invertible is replaced by quasi-invertible. Examples are known of rank-two valuation rings in which the maximal ideal is invertible—hence, in Nakano's result, prime cannot be replaced by maximal.

**Lemma.** If $P$ is an invertible prime ideal in $R$ then $\cap_n P^n$ is a prime ideal.

**Proof.** The proof is the same as that of the first part of Theorem 4 of [1].

**Theorem.** If every proper prime ideal of $R$ is quasi-invertible, then $R$ is a Dedekind domain.

**Proof.** If $R$ is a field there is nothing to prove. Let $M$ be an arbitrary proper maximal ideal of $R$ and denote by $R_M$ the quotient ring of $R$ with respect to $M$ (see [3, pp. 218–228]). Let $N$ denote the ideal consisting of the elements $x \in R$ such that there exists an element $m \in M$ such that $mx = 0$. Let $h$ be the natural homomorphism from

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$R$ onto $\bar{R} = R/N$. If $T$ and $\bar{T}$ denote the total quotient rings of $R$ and $\bar{R}$, respectively, then $h$ may be continued to a homomorphism $f$ from $T$ into $\bar{T}$ (where $f(a/b) = h(a)/h(b)$). We may suppose that $\bar{R} \subseteq R_M \subseteq \bar{T}$ (see [3]).

Since $M$ is a quasi-invertible maximal ideal in $R$, then $MM^{-1} = R$. Therefore $\bar{R} = f(MM^{-1}) = f(M)f(M^{-1}) = h(M)f(M^{-1})$, and, hence, $R_M = h(M)R_M f(M^{-1})R_M$. It follows that the maximal ideal $h(M)R_M$ in $R_M$ is invertible. By the lemma, $\bigcap_n [h(M)R_M]^n$ is a prime ideal in $R_M$, and, therefore, there exists a prime ideal $P \subseteq M \subseteq R$ such that $h(P)R_M = \bigcap_n h(M^n)R_M$. Suppose $P \neq (0)$. If $PP^{-1} \subseteq M$ then $h(P)R_M = R_M$ and it follows that the prime ideal $h(P)R_M$ is invertible. If $PP^{-1} \subseteq M$, then there exists a positive integer $n$ such that $PP^{-1} \subseteq M^n$ and $PP^{-1} \subseteq M^{n+1}$ since $PP^{-1} > P$. Since $M$ is invertible, there exists an ideal $Q$ in $R$ such that $QM^n = PP^{-1}$ and $Q \subseteq M$. Hence $h(P)R_M = h(Q)R_M = h(Q)h(M^n)R_M = h(M^n)R_M$, and therefore, $h(P)R_M = f(P)R_M f(P^{-1})R_M = h(P)R_M f(P^{-1})R_M = h(M^n)R_M$. This implies that the prime ideal $h(P)R_M$ is invertible. Thus, in either case, the prime ideal $h(P)R_M$ is invertible, which is a contradiction—it is clear that an invertible proper prime cannot properly contain another invertible prime. It follows that $P = (0)$ and therefore $R_M$ is a domain in which proper prime ideals are maximal. Hence proper prime ideals are maximal in $R$ and therefore proper prime ideals are invertible in $R$. It follows that $R$ is a Dedekind domain (see [1]).

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