2. Let $\hat{H}^n(U, A) \cong \hat{H}^n(U, B)$ in two successive dimensions and for all subgroups but do not assume the isomorphisms induced by a module homomorphism. It would not be reasonable to expect isomorphisms for all n and all subgroups. The following counterexample justifies our pessimism. Let $G = G_p(a, b: a^3 = b^7 = 1, aba^{-1} = b^2)$; let A be Z with trivial action and B the result of dimension shifting down two steps. Then $\hat{H}^q(G, A; 7) = \hat{H}^{q-2}(G, B; 7) = 0$ for $q = 1, 2, 3, 4, 5$ and $\hat{H}^6(G, A; 7) = \hat{H}^4(G, B; 7) \neq 0$.

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QUASI-INVERTIBLE PRIME IDEALS

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In this note R will denote a commutative ring with unit and a proper ideal of R is an ideal of R different from (0) and R. Nakano has shown that R is a Dedekind domain, provided that every proper prime ideal of R is invertible [1]. In [2], Krull defines a prime ideal P to be quasi-invertible provided $PP^{-1} > P$, where $>$ denotes proper containment and P^{-1} is the set of elements x in the total quotient ring of R such that $xP \subset R$. The purpose of this note is to prove that Nakano's result remains valid when invertible is replaced by quasi-invertible. Examples are known of rank-two valuation rings in which the maximal ideal is invertible—hence, in Nakano's result, prime cannot be replaced by maximal.

LEMMA. *If P is an invertible prime ideal in R then $\bigcap_n P^n$ is a prime ideal.*

PROOF. The proof is the same as that of the first part of Theorem 4 of [1].

THEOREM. *If every proper prime ideal of R is quasi-invertible, then R is a Dedekind domain.*

PROOF. If R is a field there is nothing to prove. Let M be an arbitrary proper maximal ideal of R and denote by R_M the quotient ring of R with respect to M (see [3, pp. 218–228]). Let N denote the ideal consisting of the elements $x \in R$ such that there exists an element $m \notin M$ such that $mx = 0$. Let h be the natural homomorphism from

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R onto $\bar{R}=R/N$. If T and $\bar{T}$ denote the total quotient rings of R and $\bar{R}$, respectively, then h may be continued to a homomorphism f from T into $\bar{T}$ (where $f(a/b)=h(a)/h(b)$). We may suppose that $\bar{R}\subset R_M\subset\bar{T}$ (see [3]).

Since M is a quasi-invertible maximal ideal in R, then $MM^{-1}=R$. Therefore $\bar{R}=f(MM^{-1})=f(M)f(M^{-1})=h(M)f(M^{-1})$, and, hence, $R_M=h(M)R_Mf(M^{-1})R_M$. It follows that the maximal ideal $h(M)R_M$ in R_M is invertible. By the lemma, $\bigcap_n [h(M)R_M]^n$ is a prime ideal in R_M, and, therefore, there exists a prime ideal $P\subset M\subset R$ such that $h(P)R_M=\bigcap_n h(M^n)R_M$. Suppose $P\neq(0)$. If $PP^{-1}\not\subset M$ then $h(PP^{-1})R_M=R_M$ and it follows that the prime ideal $h(P)R_M$ is invertible. If $PP^{-1}\subset M$, then there exists a positive integer n such that $PP^{-1}\subset M^n$ and $PP^{-1}\not\subset M^{n+1}$ since $PP^{-1}>P$. Since M is invertible, there exists an ideal Q in R such that $QM^n=PP^{-1}$ and $Q\not\subset M$. Hence $h(PP^{-1})R_M=h(QM^n)R_M=h(Q)h(M^n)R_M=h(M^n)R_M$, and therefore, $h(PP^{-1})R_M=f(PP^{-1})R_M=f(P)R_Mf(P^{-1})R_M=h(P)R_Mf(P^{-1})R_M=h(M^n)R_M$. This implies that the prime ideal $h(P)R_M$ is invertible. Thus, in either case, the prime ideal $h(P)R_M$ is invertible, which is a contradiction—it is clear that an invertible proper prime cannot properly contain another invertible prime. It follows that $P=(0)$ and therefore R_M is a domain in which proper prime ideals are maximal. Hence proper prime ideals are maximal in R and therefore proper prime ideals are invertible in R. It follows that R is a Dedekind domain (see [1]).

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