PERFECT NULL SETS IN COMPACT HAUSDORFF SPACES

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Our purpose is to establish the following

THEOREM. *Suppose μ is a nontrivial regular nonatomic Borel measure on a compact Hausdorff space X. Then X contains a perfect set P such that $\mu(P)=0$.*

To this end, we shall use two theorems of W. Rudin [1]:

[R_1]. If Q is a compact Hausdorff space without perfect sets and $f \in C(Q)$, then $f(Q)$ is countable.

[R_2]. If Q is a compact Hausdorff space without perfect sets, m is a regular Borel measure on Q and if $m(E)=0$ for every set E which consists of a single point, then $m(Q)=0$ (i.e., m vanishes identically).

PROOF. By [R_2], there exists a perfect subset A of X. If $\mu(A)=0$, we are through; otherwise, considering the restriction of μ to A, it suffices to suppose that X is perfect. It then follows from a proof of Urysohn's Lemma that there exists a continuous function f from X onto the real interval $[0, 1]$. Let $\{C_\lambda\}$ be an uncountable collection of pairwise disjoint closed perfect subsets of $[0, 1]$. The inverse images, $X_\lambda = f^{-1}(C_\lambda)$, are pairwise disjoint closed subsets of X each of which, by [R_1], contains a perfect set P_λ. Let $P = P_\lambda$ where $\mu(P_\lambda)=0$.

BIBLIOGRAPHY

1. Walter Rudin, *Continuous functions on compact sets without perfect subsets*, Proc. Amer. Math. Soc. 8(1957), 39–42.

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