AN EXAMPLE IN ČECH COHOMOLOGY

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In this note, we give an example of a compact space $X$ with integral Čech cohomology groups $H^g(X) = 0$, $g > 0$, but which can be mapped essentially onto the three-sphere $S^3$. This cannot occur for finite-dimensional $X$ [2].

We construct such an $X$ for each odd prime $p$, which we now suppose fixed. Define $X_0$ to be $S^{2p} \cup e^2p+1$, the $2p$-sphere with a $(2p+1)$-cell attached by a map of degree $p$. Inductively, we define $X_{n+1}$ to be the $(2p-2)$-fold suspension $E^{2p-2}X_n$ of $X_n$, $n \geq 0$. We also define maps $\alpha_n: X_n \to X_{n-1}$, $n \geq 1$, by $\alpha_n = E^{2p-2}\alpha_{n-1}$, where $\alpha_1$ is defined as follows: Let $\beta: S^{2p} \to S^3$ represent a generator of $\pi_{2p}(S^3; p) \approx \mathbb{Z}_p$. Then $E^{2p-2}\beta: S^{4p-2} \to S^{2p}$ admits a coextension $\beta': S^{4p-2} \cup e^{2p+1} \to S^{4p-1} \cup e^{2p+1}$. We note that $\beta$ admits an extension $\alpha_1: S^{4p-2} \cup e^{2p+1} \to S^{4p-1} \cup e^{2p+1}$. We now define $X = \operatorname{Inv} \operatorname{Lim}\{X_n, \alpha_n\}$. It is evident that $H^g(X) = 0$, $g > 0$. The composites $f_n = \alpha_n \alpha_{n-1} \cdots \alpha_1: X_n \to S^3$ define a map $f: X \to S^3$. The proof that $f$ is essential depends on the following result of Toda [5], [1].

**Theorem [Toda].** Each $f_n$ is an essential map. Further, all suspensions of $f_n$ are essential.

Since $[X, S^3]$, the set of homotopy classes of maps of $X \to S^3$, is equal to $\operatorname{Dir} \operatorname{Lim}\{[X_n, S^3], \alpha_n^*\}$ [3, p. 228], $f$ is essential.

$X$ has the further property that $E^{n(2p-2)}X = X$, $n > 0$. The theorem of Toda implies that each $E^{n(2p-2)}f: E^{n(2p-2)}X = X \to S^{3+n(2p-2)}$ is also essential.

**Bibliography**


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