A NOTE ON ADDITION CHAINS

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A sequence of integers \(1 = a_0 < a_1 \cdots < a_r = n\), is called an addition chain for \(n\), if \(a_i = a_j + a_k\) for \(1 \leq i \leq r; 0 \leq j, k < i\). For a given \(n\), the least \(r\) for which such a chain exists is called \(l(n)\).

Scholz [3] conjectured:

\[
(1) \quad l(2^q - 1) \leq 1(q) + q - 1, \quad q \geq 1.
\]

A. Brauer [1] proved (1), provided there is a minimal chain \(\{a_i\}_{i=1}^{l(q)}\) for \(q\) such that \(a_i = a_{i-1} + a_t\), \(0 < i \leq l(q), 0 \leq t \leq i - 1\). Gioia, Subbarao, and Sugunamma [2] employ eight lemmas to prove (1) if:

\[
(2) \quad q = 2^{c_1} + 2^{c_2} + 2^{c_3} \quad c_1 > c_2 > c_3 \geq 0.
\]

Lemma 4 of [2] states that, if (2) holds, \(l(q) = c_1 + 2\).

It is observed here that (1), subject to (2) follows immediately from this lemma and Brauer's result, since

\[1, 2, 4, \cdots, 2^{c_1}, 2^{c_1} + 2^{c_2}, 2^{c_1} + 2^{c_2} + 2^{c_3}\]

is a minimal chain for \(q\) which satisfies Brauer's condition.

REFERENCES


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