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# A NOTE ON THE HAUSDORFF MOMENT PROBLEM

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In [1, pp. 630–635], J. H. Wells presented a solution of the Hausdorff moment problem for the case of a quasicontinuous mass function. The purpose of this note is to extend that result to include Riemann-integrable mass functions.

If $\{d_n\}$ is a number sequence, let $A_{np}=\binom{n}{p}\Delta^{n-p}d_p$, $n\geqq p$, $p=0, 1, 2, \cdots$. We observe that [1, p. 634, Theorem 2.4(ii)(b)] may be stated as follows:

If $\epsilon>0$, there is a finite collection $C$ of nonoverlapping subsegments $(u, v)$ of the segment $(0, 1)$ such that $\sum_C (v-u)=1$ and if $u<y<z<v$, then there is a positive integer $N$ such that if $n>N$, $\left|\sum_{nv<p\leq nz} A_{np}+\sum_{nu\leq p<ny} A_{np}\right|<\epsilon$.

The arguments used to establish [1, p. 634, Theorem 2.4] and the associated theorems and lemmas [1, pp. 630–633] are readily modified to supply a proof of the following theorem.

THEOREM. *If $\{d_n\}$ is a number sequence, the following two statements are equivalent:*

(i) *There is a function $g$ Riemann-integrable on $[0, 1]$ such that $d_n=\int_{[0,1]}I^n dg$, $n=0, 1, 2, \cdots$;*

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(ii) (a) *there is a number $M$ such that* $\left|\sum_{p=0}^{k} A_{np}\right| < M$, $0 \leqq k \leqq n$, $n = 0, 1, 2, \cdots$, *and*

(b) *if* $\epsilon > 0$ *and* $0 < \delta < 1$, *there is a finite collection $C$ of nonoverlapping subsegments* $(u, v)$ *of the segment* $(0, 1)$ *such that* $\sum_C (v-u) > 1-\delta$ *and if* $u < y < z < v$, *then there is a positive integer $N$ such that if* $n > N$,

$$\left| \sum_{nv < p \leq nz} A_{np} + \sum_{ny \leq p < nz} A_{np} \right| < \epsilon.$$

The crux of the matter lies in the observation that [1, p. 633, Lemma 2.3] holds if the mass function is Riemann-integrable on [0, 1], and[1] in noticing the following Ascoli-type result (compare with [1, p. 630, Theorem 2.1]):

LEMMA. *Suppose* $\{f_n\}$ *is a uniformly bounded infinite sequence of real functions from* $[0, 1]$ *and if* $\epsilon > 0$ *and* $0 < \delta < 1$, *there is a finite collection $C$ of nonoverlapping subsegments* $(u, v)$ *of the segment* $(0, 1)$ *such that* $\sum_C (v-u) > 1-\delta$ *and if* $u < y < z < v$, *then there is a positive integer $N$ such that if* $n > N$,

$$|f_n(y) - f_n(z)| < \epsilon,$$

*and* $\{g_n\}$ *is an infinite subsequence of* $\{f_n\}$ *which converges at each point of a countable set which is dense in* $[0, 1]$. *If, for each $x$ in* $[0, 1]$, $h(x)$ *is a cluster point of* $\{g_n(x)\}$, *then on* $[0, 1]$ $h$ *is Riemann-integrable and* $\{g_n\}$ *converges almost everywhere to* $h$.

## REFERENCE

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[1] The author is indebted to the referee for suggesting that the lemma be stated in the paper.