that to every sequence $\alpha_1, \alpha_2, \cdots$ (with $\alpha_1 = 1$) of nonnegative integers there exists exactly one real number in the interval $[0, 1)$ to which the given sequence $\alpha_1, \alpha_2, \cdots$ corresponds.

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A SHORT PROOF OF AN INEQUALITY
FOR THE PERMANENT FUNCTION

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Let $A$ be a substochastic matrix, i.e., a square matrix of nonnegative numbers with each row sum no greater than 1. We have obtained a lower bound for the permanent of $I - A$.

Theorem. If $A$ is a substochastic matrix, then

$$\text{per} (I - A) \geq 0.$$ 

It was brought to our attention by Marcus and Minc [2] that Brualdi and Newman have proved this theorem. Indeed, two proofs of this theorem are contained in a paper that will appear in the Oxford Quarterly [1]. The proof that we shall give, shorter than and quite different from the Brualdi-Newman proofs, shows that this theorem is almost a corollary of the Ryser representation of the permanent.

Let $B$ be an $n$-square matrix and let $B_r$ denote a matrix obtained from $B$ by replacing some $r$ columns of $B$ by zero columns. Let $S(B_r)$ be the product of the row sums of the matrix $B_r$. Ryser [3] has proved that the permanent of $B$ is given by

$$\text{per} (B) = S(B_0) + \sum (-1)^r S(B_r) + \sum (-1)^2 S(B_2) + \cdots + \sum (-1)^{n-1} S(B_{n-1}),$$

where $\sum (-1)^r S(B_r)$ denotes the sum over all $\binom{n}{r}$ replacements of $r$ of the columns of $B$ by zero columns.

Let $B = I - A$ where $A$ is a substochastic matrix. The $i$th row sum of $B_r$ is nonpositive or nonnegative according to whether the $i$th column of $B_r$ is a zero or a nonzero column. Hence there are at least $r$ row sums of $B_r$ that are nonpositive and at least $n - r$ that are nonnegative. Therefore

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We are indebted to Morris Newman for a preprint of [1]. Morris Newman informs me that essentially the same proof was communicated to him independently by Hazel Perfect.

References