LOCAL FLATNESS OF COMBINATORIAL
MANIFOLDS IN CODIMENSION ONE

WILLIAM A. LABACH

We derive here a fundamental theorem of Brown [1] from a theorem of Cairns [2].

Theorem. If \( K \) is a combinatorial \( n \)-manifold without boundary rectilinearly embedded in \( \mathbb{R}^{n+1} \) then \( K \) is locally flat in \( \mathbb{R}^{n+1} \).

Proof. Let \( x \) be any point of \( K \) and let \( v \) be a vertex of \( K \) containing \( x \) in the interior of its star, \( \text{St}(v, K) \). Without loss of generality, we may assume that \( v \) is the origin in \( \mathbb{R}^{n+1} \). The radial projection \( \Gamma \) of the link, \( \text{Lk}(v, K) \), of \( v \) in \( K \) on \( S^n \) is a combinatorial \((n - 1)\)-sphere in \( S^n \) whose cells are geodesic simplexes on \( S^n \). By the main theorem of [2], there is a homeomorphism \( h \) of \( S^n \) (onto itself) taking \( \Gamma(\text{Lk}(v, K)) \) onto \( S^{n-1} \). Let \( h^* \) denote the radial extension of \( h \) to a homeomorphism of \( \mathbb{R}^{n+1} \). Then \( h^* \) maps \( \text{St}(v, K) \) into \( \mathbb{R}^n \). Thus \( K \) is locally flat in \( \mathbb{R}^{n+1} \).

Bibliography


University of Illinois and
Northwestern University

Received by the editors August 25, 1965.

1 This work was contained in the author's doctoral thesis written at the University of Illinois under the direction of Professor S. S. Cairns and was partially supported by the National Science Foundation under Grant No. GP-730.