The Koebe function \( z/(1+z)^2 \) is positive everywhere on \(|z|=1\), \( z \neq -1 \), and lies in the Hardy class \( H^p \) for every \( p < 1/2 \). We show that this behavior is extreme by proving the following

**Theorem.** If \( f(z) \in H^{1/2} \) and \( |f(z)| \geq 0 \) a.e. on \(|z|=1\) then \( f(z) \) is a constant.

**Proof.** We may assume that \( f(z) \) is not identically 0. If \( B(z) \) denotes the Blaschke product for the zeros of \( f(z) \) then, as usual, we can write

\[
1 \quad f(z) = B(z)F^2(z), \quad F(z) \in H^1.
\]

We write the condition \( f(z) \geq 0 \) as the equation \( f(z) = |f(z)| \) and conclude from (1) that

\[
2 \quad B(z)F^2(z) = |F^2(z)| \quad \text{a.e. on } |z| = 1.
\]

Since \( f(z) \) is not identically 0 it follows that \( F(z) \) is nonzero a.e. on \(|z|=1\). Thus we may divide (2) by \( F(z) \) and obtain

\[
3 \quad B(z)F(z) = \overline{F(z)} \quad \text{a.e. on } |z| = 1.
\]

But the left side of (3) is \( H^1 \) and so has all negative Fourier coefficients 0, the right side is conjugate \( H^1 \) and so has all positive Fourier coefficients 0!.

Thus only the constant term remains and we conclude that both sides are constants. This is to say \( B(z)F(z) \) and \( F(z) \) are both constants and so indeed \( f(z) = (B(z)F(z)) \). \( F(z) \) is a constant.

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