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GROUPS AND SEMIGROUPS WITH SOLVABLE WORD PROBLEMS

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This note gives a direct proof of the following theorem of M. O. Rabin [**1**, Theorem **4**]:

THEOREM. *A finitely generated group has a solvable word problem (with respect to a given system of generators) if and only if it is computable.*

A group is said [**1**] to have a *solvable word problem* with respect to a finite set of generators $\{g_1, \cdots, g_n\}$ if one can effectively determine whether or not two words on $\{\sigma_1, \cdots, \sigma_n\}$ represent the same group element. (Rabin's definition, though equivalent for groups, prevents the theorem from applying to semigroups. For convenience we suppose that the inverse of a generator is a generator.) A group is said to be *computable* if it has a *recursive realization* $\{S, x\}$—i.e. it is isomorphic to the group formed by a recursive subset S of the positive integers and a recursive function $x(i, j)$ on S that satisfies the group multiplication axioms.

PROOF. Consider some recursive realization, $\{S, x\}$ of a given finitely generated computable group. Let s_i be the element of S corresponding to the generator g_i. For any word $W=\sigma_a\sigma_b\cdots\sigma_z$ one can effectively compute the integer

$$w = s_a \times s_b \times \cdots \times s_z.$$

As two such words, W and W^1, are equivalent if and only if $w=w^1$,

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our group has a solvable word problem with respect to the given set of generators. Conversely suppose that a group has a solvable word problem with respect to *some* finite set of generators $\{g_1, \cdots, g_m\}$. One can effectively list the words on $\{\sigma_1, \cdots, \sigma_m\}$ lexicographically; since the word problem is solvable one can even do this *omitting any word that represents the same group element as some word already listed*: $W_1, W_2, W_3, \cdots$. Let S be the set of indices in this list (it is either the positive integer or a finite segment thereof). For any i and j define $x(i, j)$ as:

$$(\mu k)\{W_k \text{ represents the same group element as } W_iW_j\}.$$

Clearly $\{S, x\}$ is a recursive realization of our group.

REMARK 1. One can replace "group" by "semigroup" throughout the above theorem and its proof.

REMARK 2. As noted in [1], our theorem implies that "having a solvable word problem" is a property of the group and not of the manner in which it is presented.

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