A CONTACT STRUCTURE FOR ODD DIMENSIONAL SPHERICAL SPACE FORMS[1]

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Let $S^{2n-1} \subset R^{2n}$ denote an odd dimensional sphere of constant curvature $K>0$, embedded as the sphere of radius $K^{-1/2}$ in euclidean space. According to Gray [1] the linear differential form $\omega = \sum_{i=1}^{n}(x^i dx^{n+i} - x^{n+i} dx^i)$ defines a contact structure on S^{2n-1}, i.e. the restriction satisfies $\omega \wedge (d\omega)^{n-1} \neq 0$. The unitary group $U(n)$ is embedded in the orthogonal group $O(2n)$ as the set of all block form matrices

$$\begin{pmatrix} A & B \\ -B & A \end{pmatrix}$$

such that ${}^tA \cdot A + {}^tB \cdot B = I$ and ${}^tA \cdot B = {}^tB \cdot A$. Now it is straightforward to check that $U(n)$ is the subgroup of all elements of $O(2n)$ which preserve ω. Let Γ be a finite subgroup of $O(2n)$ in which only the identity element has $+1$ for an eigenvalue. Then it follows from ([2], remarks on p. 155, last column of chart on p. 208) that Γ is conjugate in $O(2n)$ to a subgroup of $U(n)$. On the other hand the $(2n-1)$-dimensional complete connected riemannian manifolds of constant curvature $K>0$ are just the riemannian quotient manifolds S^{2n-1}/Γ with Γ given as above. In summary, we have observed:

THEOREM. *Let M be a complete connected riemannian manifold of odd dimension $2n-1$ and constant curvature $K>0$. Then M inherits a contact structure from the linear differential form $\sum_{i=1}^{n}(x^i dx^{n+i} - x^{n+i} dx^i)$ on S^{2n-1}.*

REFERENCES

1. J. W. Gray, *Some global properties of contact structures*, Ann. of Math. **69** (1959), 421–450.

2. J. A. Wolf, *Spaces of constant curvature*, McGraw-Hill, New York, 1967.

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