A CONTACT STRUCTURE FOR ODD DIMENSIONAL SPHERICAL SPACE FORMS

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Let $S^{2n-1} \subseteq R^{2n}$ denote an odd dimensional sphere of constant curvature $K > 0$, embedded as the sphere of radius $K^{-1/2}$ in euclidean space. According to Gray [1] the linear differential form $\omega = \sum_{i=1}^{n} (x^i dx^{n+i} - x^{n+i} dx^i)$ defines a contact structure on $S^{2n-1}$, i.e. the restriction satisfies $\omega \wedge (d\omega)^{n-1} \neq 0$. The unitary group $U(n)$ is embedded in the orthogonal group $O(2n)$ as the set of all block form matrices

\[
\begin{pmatrix}
  A & B \\
  -B & A 
\end{pmatrix}
\]

such that $^tA \cdot A + ^tB \cdot B = I$ and $^tA \cdot B = ^tB \cdot A$. Now it is straightforward to check that $U(n)$ is the subgroup of all elements of $O(2n)$ which preserve $\omega$. Let $\Gamma$ be a finite subgroup of $O(2n)$ in which only the identity element has $+1$ for an eigenvalue. Then it follows from ([2], remarks on p. 155, last column of chart on p. 208) that $\Gamma$ is conjugate in $O(2n)$ to a subgroup of $U(n)$. On the other hand the $(2n-1)$-dimensional complete connected riemannian manifolds of constant curvature $K > 0$ are just the riemannian quotient manifolds $S^{2n-1}/\Gamma$ with $\Gamma$ given as above. In summary, we have observed:

**Theorem.** Let $M$ be a complete connected riemannian manifold of odd dimension $2n-1$ and constant curvature $K > 0$. Then $M$ inherits a contact structure from the linear differential form $\sum_{i=1}^{n} (x^i dx^{n+i} - x^{n+i} dx^i)$ on $S^{2n-1}$.

**References**


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