KILLING KNOTS

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Let \( k \) be a tame knot in \( S^3 \), \( N(k) \) a regular neighborhood of \( k \) and \( M = S^3 \setminus \text{Int } N(k) \). \( M \) may be collapsed \([1]\) to a 2-complex \( K \). Let \( D \) be a disk in \( N(k) \) with \( D \cap \partial N(k) = \partial D \) and \( \partial D \) not contractible on \( \partial N(k) \). \( M = N(K) \) is the mapping cylinder \( C_f \) of a map \( f: \partial N(K) \rightarrow K \), so \( D \cup C_f \cup K = L \) is a spine for \( M \cup N(D) = U \). \( \partial U \) is a 2-sphere, so \( U \) is a cell and \( L \) is cellular. Then \( S^3 \setminus L \equiv S^3 \). \( k \cap L \) is a single point in the interior of \( D \) and \( k \cap (S^3 \setminus \text{Int } U) \) is unknotted in \( S^3 \setminus \text{Int } U \). It is easy to see that

**Theorem 1.** If \( k \) is a tame knot in \( S^3 \), there is a cellular 2-complex \( L \) in \( S^3 \) such that \( k \cap L \) is a single point, and under the projection \( \pi: S^3 \rightarrow S^3 \setminus L \equiv S^3 \), \( \pi(k) \) is a tame and unknotted simple closed curve.

Note that if \( k \cap L = \emptyset \), then \( \pi(k) \) is unknotted if and only if \( k \) is unknotted.

**Theorem 2.** If \( G = \pi_1(S^3 \setminus k) \) is a knot group, there is a metric \( d \) on \( E^3 \) (inducing the standard topology) and a closed set \( P \) homeomorphic to \( E^1 \) such that \( P \) is "straight" (for any three points \( x, y, z \in P \), \( d(x, y) = d(y, z) = d(x, z) \)) and \( \pi_1(E^3 \setminus P) = G \).

**Proof.** It may be assumed that \( k \cap U = k \cap N(D) \) is a straight line segment. Let \( P = k \cap \text{Int } U \). \( \text{Int } U = E^3 \) and inherits its metric from \( S^3 \). It is clear from the construction that \( \pi_1(\text{Int } U \setminus P) = \pi_1(S^3 \setminus k) \).

**Reference**


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