A SIMPLE PROOF OF A WELL-KNOWN
OSCILLATION THEOREM

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Theorem ([1], [2]). The equation

\[(ry')' + py = 0 \quad (r > 0, \ r \text{ and } p \text{ continuous})\]

is oscillatory on \([0, +\infty)\) provided

\[\int_{0}^{\infty} \frac{1}{r} = \int_{0}^{\infty} p = +\infty.\]

Proof. If (1) is nonoscillatory, the Riccati equation

\[z' + \frac{z^2}{r} + p = 0\]

has a solution on some half-line \([a, \infty)\); thus, for large \(t\),

\[z(t) + \int_{a}^{t} \frac{z^2}{r} = z(a) - \int_{a}^{t} p < 0.\]

Let \(R(t) = \int_{a}^{t} \frac{z^2}{r}\); (4) says that

\[R^2 \leq R' \cdot r\]

for \(t \geq b > a\) (\(b\) sufficiently large). Separation of variables and integration of (5) give

\[\int_{b}^{t} \frac{1}{r} \leq R^{-1}(b) - R^{-1}(t) \leq R^{-1}(b), \quad t \geq b,\]

which contradicts (2).

References


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