INDEPENDENCE OF A CERTAIN AXIOMATIC SYSTEM

LINO GUTIERREZ-NOVOA

To prove the independence of the system of axioms introduced in [1] we exhibit here ten models, each of them satisfying all the axioms but one; e.g. model $M_5$ fails to satisfy $P_5$ of [1].

We call model $\Gamma$ the Euclidean 3-space with the usual vector structure, introduce a suitable order function $\phi$ and the usual notion of orthogonality. Let $A_i (i = 1, 2, 3, 4)$ be the position vectors of four points and define:

$$\phi(A_1, A_2, A_3, A_4) = \text{sign det } | A_2 - A_1, A_3 - A_1, A_4 - A_1 |.$$

Model $\Gamma$ shows the relative consistency of the system.

$M_1$—In $\Gamma$, take a new order function: $\phi_1 = | \phi |$.

$M_2$—In $\Gamma$, change the sign of $\phi$ for exactly one nonsingular tetrad and its opposite.

$M_3$—Let the $\phi$ of $\Gamma$ be identically 0.

$M_4$—Adjoin one point $X$ to $\Gamma$ and extend $\phi$: $\phi(X, A_1, A_2, A_3) = \phi(O, A_1, A_2, A_3)$ where $O$ is the origin and $A_i \subseteq \Gamma$.

$M_5$—Vectors of $\Gamma$ with integral components and $\phi$ restricted accordingly.

$M_6$—In the hyperbolic space $H^3$ take any orientation function for $\phi$ and keep the usual orthogonality notion.

$M_7$—Take the vectors of $\Gamma$ with rational components and restrict $\phi$.

$M_8$—Imbed $\Gamma$ in the projective space $P^3$ adding the ideal plane $\Omega$. Let $\gamma$ be a real conic on $\Omega$ and define $l \perp \pi$ (in $\Gamma$) to mean $l \cap \Omega$ and $\pi \cap \Omega$ are pole and polar with respect to $\gamma$.

$M_9$—Same as before but $l \perp \pi$ is defined only if $l \cap \Omega$ is a two-tangent point with respect to $\gamma$.

$M_{10}$—Let $\gamma$ be an elliptic correlation, not a polarity, on $\Omega$, and define $l \perp \pi$ accordingly.

The proof is now complete.

REFERENCE


UNIVERSITY OF ALABAMA

Received by the editors May 13, 1968.