Let $\mathcal{J}$ denote the Banach space of trace class operators on a complex Hilbert space $H$, in the norm $||T||_1 = \text{Tr}(|T|)$. The space $\mathcal{J}$ is a two-sided ideal in the algebra $\mathcal{L}$ of all bounded operators on $H$. See [4].

**Theorem.** If $\Phi$ is a linear isometry of the Banach space $\mathcal{J}$ onto itself, then there exists a $^*$-automorphism or a $^*$-antiautomorphism $\alpha$ of $\mathcal{L}$ and a unitary operator $U$ in $\mathcal{L}$ such that $\Phi(T) = \alpha(TU)$, ($T$ in $\mathcal{J}$).

**Remark 1.** The theorem provides a partial answer to [3, Remark 1, p. 231].

**Proof.** The adjoint $\Phi'$ is a linear isometry of $\mathcal{L}$ onto $\mathcal{L}$ so by results of Kadison [2, Theorem 7, Corollary 11] has the form $\Phi'(A) = U\alpha(A)$ where $\alpha$ and $U$ are as described in the statement of the theorem. It is elementary that $\Phi(T) = \Psi(TU)$ where $\Psi' = \alpha$. The proof will be complete if it is shown that $\alpha$ is the adjoint of $\alpha^{-1}$ (restricted to $\mathcal{J}$). By the folk result [1, pp. 256, 9] it is sufficient to check this in the following two cases:

(i) $\alpha(A) = VA V^{-1}$ with $V$ a fixed unitary operator; then $\langle T, \alpha(A) \rangle = \langle T, VA V^{-1} \rangle = \langle V^{-1}TV, A \rangle = \langle \alpha^{-1}(T), A \rangle$.

(ii) after the choice of an orthonormal basis, $\alpha(A)$ is the transposed matrix of $A$; then $\langle T, \alpha(A) \rangle = \text{Tr}(T\alpha(A)) = \text{Tr}(\alpha(T)A) = \langle \alpha^{-1}(T), A \rangle$.

**Remark 2.** A previous version of the above proof exploited a knowledge of the extreme points of the unit sphere of $\mathcal{J}$. These were determined to be the partial isometries with initial (hence final) domain one-dimensional.

**References**


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Received by the editors March 7, 1969.

1 This research was supported by the National Science Foundation Grant GP-8291.