THE TANGENT BUNDLE OF THE LONG LINE

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We prove the following surprise.

THEOREM. *The tangent bundle of the long line is not trivial.*

PROOF. The long line L is defined in Hocking and Young [1], for example. It is easy to check that L, with the topology defined there, supports the structure of a C^∞ manifold. In fact, it is proved in [2] that L (there called the Alexandroff Half Line) can be made into a real analytic manifold (I thank the referee for this reference). Let us recall what it means for a real vector bundle E to be trivial. The bundle $E \xrightarrow{\pi} X$ is trivial if there is a bundle map $\phi: E \to \xi^n$ covering the identity: $X \to X$, where ξ^n is the trivial $\mathbf{R}^n$ bundle over X and n is the dimension of the fibre of E. But this is easily seen to be equivalent to the existence of a bundle map $\tilde{\phi}: E \to \mathbf{R}^n$ where $\mathbf{R}^n$ is considered as a bundle over a point $*$. (If $\omega: \xi^n \to \mathbf{R}^n$ is projection onto the coordinate of each fibre, then $\tilde{\phi} = \omega \circ \phi$ is such a map. If $\tilde{\phi}$ is given then $\tilde{\phi}^{-1}\mathbf{R}^n = \xi^n$ and $E \cong \tilde{\phi}^{-1}\mathbf{R}^n$ where $\tilde{\phi}^{-1}$ means the pull back under $\tilde{\phi}$.) So let TL be the tangent bundle of L and ϕ be a C^∞ trivialization

$$\begin{array}{ccc} TL & \xrightarrow{\phi} & \mathbf{R}^1 \\ \pi \downarrow & & \downarrow \\ L & \longrightarrow & *. \end{array}$$

Then $g(x, y) = \phi(x) \cdot \phi(y)$ defines a C^∞ positive definite inner product on the fibres of TL (ϕ is an isomorphism on each fibre). Thus L would be a Riemannian manifold. But it is well known (see Kobayashi-Nomizu [3, p. 166]) that a Riemannian manifold (not necessarily assumed paracompact) has a metric space structure which defines the manifold topology. Now it is well known that L is not paracompact, and hence is not a metric space. This contradiction proves the theorem.

REFERENCES

1. J. G. Hocking and G. S. Young, *Topology,* Addison-Wesley, Reading, Mass., 1961.

2. H. Kneser, *Analytische Struktur und Abzählbarkeit,* Ann. Acad. Sci. Fenn. Ser. A.I. No. 251/5 (1968), 8 pp.

3. S. Kobayashi and K. Nomizu, *Foundations of differential geometry.* Vol. I, Interscience, New York, 1963.

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