THE INTERSECTION OF INDECOMPOSABLE CONTINUA

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The theme of this paper is to reveal a fundamental result concerning the 2-finished sum of compact continua, one of which is hereditarily indecomposable.

DEFINITION. The set M is the k-finished sum of a set of subcontinua, $\{M_1, \cdots, M_k\}$, if and only if

$$M = \bigcup_{i=1}^{k} M_i$$

and for each fixed j, $1 \leqq j \leqq k$,

$$M_j - \bigcup_{1 \leq i \leq k;\, i \neq j} M_i \neq \emptyset.$$

In this paper we shall consider the space S to be a Moore space satisfying Axiom 0 and Axiom 1 of R. L. Moore. The boundary of a set D with respect to a set M will be denoted by $F(D)_M$.

THEOREM. *If M is the 2-finished sum of compact continua, M_1 and M_2, such that M_1 is hereditarily indecomposable and $M_1 \cap M_2 \neq \emptyset$, then there exists at least one point in $M_1 \cap M_2$ which is a limit point of both $(M_1 - M_2)$ and $(M_2 - M_1)$.*

PROOF. Since $M_1 \cap M_2 \neq \emptyset$, then M is a compact continuum. Suppose that no point of $M_1 \cap M_2$ is a limit point of both $H = M_1 - M_2$ and $K = M_2 - M_1$. That is, suppose $F(H)_M \cap F(K)_M = \emptyset$. The supposition implies that $\overline{H} \cap \overline{K} = \emptyset$.

Let T be a component of H. The reference [1] implies that there exists a point $p \in F(H)_M \cap \overline{T}$. Also it is noted that $\overline{T} \subset M_1$ since M_1 is a closed point set containing T. Since H is a domain relative to M then $p \notin H$ and $p \in M_2$.

The point $p \notin \overline{K}$ since $p \in M_1 \cap F(H)_M$. Thus the point set $(M_2 - \overline{K}) = M_2 \cap (M - \overline{K})$ is a domain relative to M_2 containing p. Let L be the component of $(M_2 - \overline{K})$ containing the point p. The reference [1] implies that there exists a point $q \in F(K)_M \cap \overline{L}$. The point $q \notin \overline{T}$ for if so then $q \in F(H)_M$ which contradicts the supposition. The point set $\overline{L} \subset M_1$ since M_1 is a closed point set containing L.

By the definition of both $\overline{T}$ and $\overline{L}$ we know that each is a subcontinuum of M_1. Then since $p \in \overline{T} \cap \overline{L}$ the point set $\overline{T} \cup \overline{L}$ is a sub-

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continuum of M_1. Since $q \notin \overline{T}$ then $\overline{L} - \overline{T} \neq \emptyset$. Also since $T \subset H$ and $\overline{L} \cap H = \emptyset$, then $\overline{T} - \overline{L} \neq \emptyset$. Thus the point set $\overline{T} \cup \overline{L}$ is a decomposable subcontinuum of M_1. This contradicts M_1 being hereditarily indecomposable, and thus the theorem is proved.

Bibliography

1. R. L. Moore, *Foundations of point set theory*, Amer. Math. Soc., Colloq. Publ., vol. 13, Amer. Math. Soc., Providence, R. I., 1962.

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