A TAUBERIAN THEOREM FOR THE
\((C, 1)(N, 1/(n+1))\) SUMMABILITY METHOD

H. P. DIKSHIT

Abstract. A Tauberian theorem is proved which infers the
\((C, 1)\) summability of a sequence associated with a formally differ-
ential Fourier series from its \((C, 1)(N, 1/(n+1))\) summability
under suitable conditions.

In the present note we answer in the affirmative a question raised
in [2, p. 19]. The definitions and notations of [2] are used in this note
without further explanation.

Theorem. If the sequence \(\{s_n\}\) is summable \((C, 1)(N, 1/(n+1))\)
and if
\[
\frac{1}{s_n} - \frac{1}{s_{n-1}} = O(n^{-\delta}), \quad (0 < \delta < 1; n \to \infty),
\]
where \(s_n^1\) is the nth \((C, 1)\) mean of \(\{s_n\}\), then \(\{s_n\}\) is \((C, 1)\) summable.

It follows from this result that Theorem A of [2] is indeed a con-
sequence of Theorem 1 of [2].

We need the following results for the proof of the theorem.

Lemma 1. If a sequence \(\{s_n\}\) is summable \((A, 1/(n+1))\) to \(s\) and
\(s_n - s_{n-1} = O(n^{-\delta}), 0 < \delta < 1\), then \(\{s_n\}\) converges to \(s\).

Lemma 1 is due to Iyengar [3, cf. Theorem I].

Lemma 2. If for \(n = 0, 1, 2, \ldots, p_n > 0\) and \(p_{n+1}/p_n \leq p_{n+2}/p_{n+1} \leq 1,\)
then
\((N, p_n)(C, 1)\) is equivalent to \((C, 1)(N, p_n)\).

While proving inclusion relations for the absolute \((C, 1)(N, p_n)\) and
\((N, p_n)(C, 1)\) methods, Das has pointed out in [1] that the cor-
responding analogues for ordinary summability also hold. Thus we
have Lemma 2 as an analogue of Theorem 5 of [1].

To prove the theorem we first observe that the hypotheses of
Lemma 2 are satisfied if \(p_n = 1/(n+1)\) and therefore the summabil-
ity \((C, 1)(N, 1/(n+1))\) of \(\{s_n\}\) is the same as the \((N, 1/(n+1))\) sum-
mability of the sequence of \((C, 1)\) means of \(\{s_n\}\). These means
satisfy the hypothesis of Lemma 1 and the theorem follows.

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References


University of Allahabad, Allahabad, India and
University of Jabalpur, Jabalpur, India