CORRECTION TO A THEOREM OF MINE

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Abstract. One of the theorems published by me in a previous paper turned out to be incorrect. That theorem is replaced in this note by a corrected one.

In 1966 I published the following theorem [4, Theorem 3, p. 882]:
(In what follows $C_n$ denotes an $n$-dimensional conformally-flat Riemannian space and $C'_n$ a $C_n$ of class one.)

"The coordinates of any $C'_n$ may be so chosen that its metric assumes the normal form

$$ds^2 = \sum_i (dx^i)^2/[f(\theta)]^2, \quad \theta = \sum_i (x^i)^2,$$

where $f$ is any real analytic function of $\theta$ subject to the restriction

$$(n - 1)f' + \theta f'' - (n - 1)\theta f'^2 \neq 0, \quad (f' = df/d\theta, \text{etc.})$$

In a recent paper [1] G. M. Lancaster has proved that this theorem is incorrect by showing that the above metric does not cover a certain type of $C'_n$. The purpose of this note is to point out that the metric covers a type of $C'_n$ although it does not cover all $C'_n$, and also to give a correct form of the theorem. Before doing so I have to say that on checking an error in a previous paper of mine [2, equations (1.8), p. 107], the referee of the present note has drawn my attention to the fact that the restriction in the above theorem applies when $f'' \neq 0$; and at the same time he has given a straightforward solution of

$$(n - 1)f' + \theta f'' - (n - 1)\theta f'^2 = 0, \quad \text{where} \quad f'' \neq 0,$$

as

$$f(\theta) = \theta/[K_1 + (n - 2)K_2 \theta^{n-2}]^{1/(n-2)}, \quad K_1 > 0.$$ 

I heartily thank the referee for the pains and the interest he has taken in the paper. A correct form of the theorem which must replace the above theorem may then be stated as follows ($f' = df/d\theta$, etc.):

A $C_n$ $(n>3)$ with metric

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\[ ds^2 = \sum_i (dx^i)^2 / [f(\theta)]^2, \quad \theta = \sum_i (x^i)^2, \]

where \( f(\theta) \) is any real analytic function of \( \theta \) subject to the restriction that when \( f'' \neq 0 \),

\[ f(\theta) \neq \theta / [K_1 + (n - 2)K_2 \theta^{n-2}]^{1/(n-2)}, \quad K_1 > 0, \quad K_2 \text{ being constants}, \]

is a \( C^n \). The metric covers the case of space of constant curvature when \( f'' = 0 \).

That the \( C_n \) is a \( C^n \) is proved straightway by showing that it satisfies the Gauss-Codazzi equations. In fact, referring to my paper [4, p. 882] mentioned at the outset, it is not difficult to see that the \( C_n \) satisfies equations (7), namely

\[ R_{hijk} = R_{ijhk} - R_{hjik}, \quad \rho^2 \text{ and } \rho \bar{\rho} \text{ are given by equations (10), namely} \]

\[ \rho^2 = 4f'(f - \theta f'), \quad \rho \bar{\rho} = 4(ff' + \theta f'' - \theta f'^2). \]

We have only to take now the second fundamental tensor \( b_{ij} \) as

\[ b_{ij} = - \frac{1}{n-2} (R_{ij}/\rho + \bar{\rho}g_{ij}) \]

and establish the Gauss-Codazzi equations in exactly the same way as they have been done in a previous paper of mine [3, equations (3.8), (5.1)].

References


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