PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 27, Number 2, February 1971

SHORTER NOTES

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ON THE HYPERSPACE OF SUBCONTINUA OF AN ARC-LIKE CONTINUUM

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ABSTRACT. It is shown that the hyperspace of each arc-like continuum can be embedded in E^3.

W. R. R. Transue [1] in a beautiful note gave a positive answer to A. Connor's [2, p. 152] question "Can the hyperspace of subcontinua of the pseudoarc (with the Hausdorff metric) be embedded in E^3." This note extends this result to arc-like continua, i.e. inverse limits on arcs originally called snake-like continua by R. H. Bing [3].

Here $\{W, f_i\}$ will denote the inverse limit system with indexing set the nonnegative integers and with each factor space W. The associated inverse limit space will be denoted by $\lim\{W, f_i\}$. See [4, p. 87] for a discussion of inverse limits. The hyperspace of continua of a space X, denoted by $C(X)$, is studied in [5]. The closed interval $[0, 1]$ will be called I.

THEOREM. *The hyperspace of continua of the inverse limit space* $X=\lim\{I, f_i\}$ *embeds in* E^3.

PROOF. There is no loss to assume that none of the maps f_i is constant on an open set. Since a continuum in I is either a closed interval or a point, $C(I)$ will be identified with

$$D = \{(x, y, z) \subset E^3 | 0 \leqq x \leqq y \leqq 1, z = 0\}.$$

Take $F_i: D \to D$ by

$$F_i(x, y, 0) = (\min f_i(t), \max f_i(t), 0), \qquad t \in [x, y].$$

Now F_i is the natural map from $C(I)$ to $C(I)$ induced by f_i.

J. Segal [6] proved that the hyperspace of continua of the inverse limit space X is homeomorphic to $\lim\{C(I), F_i\}$. The proof of the theorem will be completed by embedding $\lim\{D, F_i\}$ in E^3. Now if each of the maps F_i could be *approximated by embeddings in* E^3 in the

Received by the editors December 8, 1969.
AMS 1970 *subject classifications*. Primary 54B20, 54C25, 54F50.
Key words and phrases. Arc-like continua, hyperspace of continua, imbedding.

sense of McCord [7, Theorem 2] then $\lim\{D, F_i\}$ embeds in E^3 by McCord's Theorem. To construct these approximations one must prove the following:

LEMMA. *If* $\epsilon > 0$, *then there is a homeomorphism* h_i *of* E^3 *onto* E^3 *such that* $\|h_i|D, F_i\| < \epsilon$.

PROOF. For each $(x, y, 0) \in D$ take

$$G_i(x, y, 0) = F_i(x, y, 0) + \left(0, 0, \frac{\epsilon}{4} \cdot \frac{x+y}{2}\right) \quad \text{(vector addition)}.$$

As f_i is not locally constant, a check will show that each point inverse of G_i is a point or a closed interval which does not separate D. Thus $G_i(D)$ is a topological disk and consequently there is a homeomorphism H_i of D onto $G_i(D)$ such that $\|H_i, G_i\| < \epsilon/4$ by Radó [8, Theorem 2.17]. Next $G_i(D)$ is tamed by an approximation theorem of R. H. Bing [9] which constructs a homeomorphism J_i of D into E^3, $\|J_i, H_i\| < \epsilon/4$ and with $J_i(D)$ a polyhedron. So there is a homeomorphism h_i of E^3 onto E^3 with $h_i|D = J_i$ since $J_i(D)$ is tame.

REMARK. With more care one can construct this embedding so that the intersection of the embedded $C(X)$ and the plane $x = y$ is exactly the set of degenerate subcontinua of X.

REFERENCES

1. W. R. R. Transue, *On the hyperspace of subcontinua of the pseudoarc*, Proc. Amer. Math Soc. **18** (1967), 1074–1075. MR **36** #5901.

2. R. H. Bing and R. J. Bean (Editors), *Topology seminar, Wisconsin*, 1965, Ann. of Math. Studies, no. 60, Princeton Univ. Press, Princeton, N. J., 1966. MR **34** #1974.

3. R. H. Bing, *Snake-like continua*, Duke Math. J. **18** (1951), 653–663. MR **13**, 265.

4. R. Engelking, *Outline of general topology*, PWN, Warsaw, 1965; English transl., North-Holland, Amsterdam; Interscience, New York, 1968. MR **36** #4508; MR **37** #5836.

5. J. L. Kelley, *Hyperspaces of a continuum*, Trans. Amer. Math. Soc. **52** (1942), 22–36. MR **3**, 315.

6. J. Segal, *Hyperspaces of the inverse limit space*, Proc. Amer. Math. Soc. **10** (1959), 706–709. MR **21** #7492.

7. M. C. McCord, *Embedding* $\mathcal{P}$*-like compacta in manifolds*, Canad. J. Math. **19** (1967), 321–332. MR **35** #3669.

8. T. Radó, *On continuous mappings of Peano spaces*, Trans. Amer. Math. Soc. **58** (1945), 420–454. MR **7**, 282.

9. R. H. Bing, *Approximating surfaces with polyhedral ones*, Ann. of Math. (2) **65** (1957), 456–483. MR **19**, 300.

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