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MAPPINGS OF INDECOMPOSABLE CONTINUA

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ABSTRACT. We show that every compact metric continuum is a continuous image of some indecomposable compact metric continuum.

Herein a continuum denotes a compact connected metric space. J. W. Rogers, Jr. has raised the questions: 'Which continua are continuous images of indecomposable continua?' and 'Is there an indecomposable continuum of which every indecomposable continuum is a continuous image?' [4]. We answer these questions.

LEMMA 1. *Every continuum irreducible between two points is a continuous image of some indecomposable continuum.*

PROOF. Let M be a continuum irreducible between two points, a and b. Let D denote the well-known indecomposable continuum which is the union of all semicircles in the closed upper half-plane with center $(1/2, 0)$ and with both endpoints in the Cantor ternary set on the x-axis, and all semicircles in the lower half-plane with center

$$((1/2)((1/3)^n + (2/3)(1/3)^n), 0)$$

for some nonnegative integer n and with both endpoints lying in the Cantor set and on or to the left of the line $x=(1/3)^n$. (A sketch can be found in [3, p. 206], [2, p. 332], or [4, p. 96], so none is included here.)

Now, let

$$D_a = \{(x, y) \in D : x \leqq 2/5\}, \qquad D_b = \{(x, y) \in D : x \geqq 3/5\},$$
$$A = \{y : (2/5, y) \in D_a\}$$

and observe that also

$$A = \{y : (3/5, y) \in D_b\}.$$

The continuum D_M is now obtained from the disjoint union of D_a, D_b, and $M \times A$ by identifying $(2/5, y) \in D_a$ with $(a, y) \in M \times A$ and identifying $(3/5, y) \in D_b$ with $(b, y) \in M \times A$, for each $y \in A$. (Intuitively speaking we have removed from D a copy of a closed interval crossed with A and spliced in a copy of $M \times A$.) The proof that D_M is indecomposable is straightforward, and is left to the reader. It de-

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pends upon the indecomposability of D and the irreducibility of M.

Now, we define $g:D_M \to M$ by

$$g(p) = a \qquad \text{for } p \in D_a,$$
$$g(p) = b \qquad \text{for } p \in D_b,$$
$$g(m, y) = m \qquad \text{for } (m, y) \in M \times A.$$

This is the desired mapping of D_M onto M.

THEOREM 2. *If S is a continuum, there exists an indecomposable continuum of which S is a continuous image.*

PROOF. Lemma 2 of [1] states that there is a continuous mapping of some continuum M irreducible between two points onto S; and by the above lemma there is a continuous surjection $f:D_M \to M$. The composition of these maps yields a continuous surjection $f:D_M \to S$. Since D_M is indecomposable, this is the desired mapping.

COROLLARY 3. *There is no continuum of which every indecomposable continuum is a continuous image.*

PROOF. By the above theorem, every continuum would be a continuous image of such a con'inuum, contradicting the principal result of [5].

COROLLARY 4. *If S is a continuum there exists an indecomposable continuum X which contains a copy of S as a retract.*

PROOF. First, the mapping of D_M onto M can be considered a retraction by identifying M with $M \times \{y\}$ for some $y \in A$. In turn, the mapping in Lemma 2 of [1] can be considered a retraction in a similar manner. Since composition of the retractions $g:D_M \to M$ and $r:M \to S$ yields a retraction, we set $X = D_M$ and the proof is complete.

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