REMARK ON SOME INTEGRALS INVOLVING PRODUCTS OF WHITTAKER FUNCTIONS

H. M. SRIVASTAVA

Abstract. It is observed that the literature contains erroneous formulas for infinite integrals involving the product of two Whittaker functions. For instance, the main result, involving Meijer's $G$-function, of K. L. Arora and S. K. Kulshreshtha's paper in these Proceedings and all its particular cases may be cited.

1. L. J. Slater states [2, p. 56, (3.7.17)]

\begin{align*}
\int_0^\infty v^{x-2-m-m'}(1 + v)^{-1}e^{(b+b')v/2}M_{k,m}(bv)M_{k',m'}(b'v) \, dv \\
= \left(\frac{\pi}{\sin \pi x}\right)e^{-(b+b')/2}M_{-k,m}(b)M_{-k',m}(b'), \quad \Re(x) > 0,
\end{align*}

where $M_{k,m}(x)$ denotes the Whittaker function.

Her proof of this formula makes use of the $\Gamma$-function integral

\begin{align*}
\int_0^\infty v^{x-1}(1 + v)^{-x} \, dv = \Gamma(x)\Gamma(y-x)/\Gamma(y),
\end{align*}

which is valid for $0 < \Re(x) < \Re(y)$. She applies it, however, without satisfying the condition $\Re(x) < \Re(y)$. Therefore the proof of (1) is invalid and indeed the result is not true. If, for instance, $b$ and $b'$ are positive, the integrand in (1) increases exponentially as $v \to \infty$, since

\begin{align*}
M_{k,m}(x) \sim C(k, m)x^{-k}e^{x/2} \quad (x \to \infty),
\end{align*}

where $C(k, m)$ is a constant depending upon $k$ and $m$.

Similar remarks would apply equally well to Slater's main formula (3.7.9) and its other special cases (3.7.10) through (3.7.13) in [2, pp. 55–56].

2. It may be of interest to observe that the recent formulas, involving Meijer's $G$-function, given by K. L. Arora and S. K. Kulshreshtha in [1]...
are based on the integral (1) and are therefore incorrect, at least for some values of the parameters involved. This can easily be verified by considering the well-known asymptotic expansions of the various special functions involved in the infinite integrals evaluated in [1]. The details are, therefore, omitted.

REFERENCES


DEPARTMENT OF MATHEMATICS, UNIVERSITY OF VICTORIA, VICTORIA, BRITISH COLUMBIA, CANADA