THE HYPERSPACE OF A PSEUDOARC IS A CANTOR MANIFOLD

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Abstract. The following theorem which was conjectured by C. Eberhart and S. B. Nadler, Jr., in [EN] is proved.

Theorem. The hyperspace of nonvoid subcontinua of a pseudoarc is a two-dimensional Cantor manifold.

1. Introduction. The hyperspace $C(X)$ of nonvoid subcontinua of a metric continuum $X$ has been investigated extensively. (We will restrict our discussion to metric continua.) It is known that $C(X)$ is always compact and arcwise connected [KE]. The basic work [S] establishes the relationship between $C(X)$ and inverse limit spaces. Inverse limit methods have yielded further properties of $C(X)$. Namely, $C(X)$ is acyclic in all dimensions [S], unicoherent [S], [N], and has dimension exceeding one for nondegenerate $X$ [EN]. By specializing $X$, much more can be said of $C(X)$. Notable works along this line are [D1] and [D2] where $X$ is locally connected. The hyperspace of an hereditarily indecomposable continuum $X$ also has been studied. See [KE], [EN], [R], [T] and [H]. The present paper concerns itself with one such hereditarily indecomposable continuum, the pseudoarc. It is known that the hyperspace of a pseudoarc is embeddable in Euclidean three-dimensional space [T], [H] and that its dimension is two [EN]. We add to the large collection of facts about $C(X)$ the theorem stated in the abstract. This theorem improves the dimension two assertion of [EN].

2. The function $\mu$. Let $X$ be a nondegenerate metric continuum and $C(X)$ be the space of all nonvoid subcontinua of $X$ with the Hausdorff metric [KU]. In [KE], Kelley noted the existence (originally due to Whitney [W]) of a real-valued function $\mu$ defined on $C(X)$ and having

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the following properties:

1. \( p \) is continuous;
2. If \( A, B \subseteq C(X) \), \( A \subseteq B \) and \( A \neq B \), then \( p(A) < p(B) \);
3. \( p(X) = 1 \);
4. \( p(\{x\}) = 0 \) for each \( x \in X \).

He proved among others that

(a) \( C(X) \) is an arcwise connected continuum;
(b) if \( X \) is hereditarily indecomposable, \( A, B \subseteq C(X) \), \( A \cap B \neq \emptyset \) and \( p(A) = p(B) \) then \( A = B \);
(c) \( X \) is hereditarily indecomposable if and only if \( C(X) \) contains a unique arc between every pair of its elements.

Suppose further that \( X \) is a pseudoarc. Then, in [R], it was observed that the space \( \mu^{-1}(t), 0 \leq t < 1 \), is a totally pathwise disconnected continuum. Subsequently, Eberhart and Nadler in [EN] observed that \( \mu^{-1}(t) \) is a continuous decomposition of \( X \) and hence, by [B1],
(d) \( \mu^{-1}(t) \) is a pseudoarc for \( 0 \leq t < 1 \) whenever \( X \) is a pseudoarc.

We now prove three lemmas which will be needed later.

**Lemma 2.1.** Suppose \( X \) is a pseudoarc and \( 0 \leq t < 1 \). Then there is a homeomorphism \( h_t : C(X) \to \mu^{-1}(t) \) such that \( h_t[0] = t \).

**Proof.** Using (d) above, we let \( h : X \to \mu^{-1}(t) \) be a homeomorphism. Define a mapping \( h \) on \( C(X) \) onto the hyperspace \( C(\mu^{-1}(t)) \) of the space \( \mu^{-1}(t) \) by \( h(A) = h(A) \) for each \( A \in C(X) \). Then \( h \) is a homeomorphism.

Let \( 2^X \) be the space of all nonvoid closed subsets of \( X \) with the Hausdorff metric, and \( 2^{2^X} \) be the space of all nonvoid closed subsets of \( 2^X \) with Hausdorff metric. Let \( \sigma : 2^{2^X} \to 2^X \) be defined by \( \sigma(\mathcal{A}) = \bigcup \{ A \in 2^X : A \in \mathcal{A} \} \), \( \mathcal{A} \in 2^{2^X} \). In [KE], it is shown that \( \sigma \) is continuous. Since \( C(\mu^{-1}(t)) \subseteq 2^{2^X} \), we let \( \sigma \) be the restriction on \( C(\mu^{-1}(t)) \). Then \( \sigma(\mathcal{A}) \) is a subcontinuum of \( X \), and thus \( \sigma(\mathcal{A}) \subseteq C(X) \). Let \( \mathcal{A} \subseteq \sigma(\mathcal{A}) \), so that by the property (2) of \( \mu \), \( \mu(A) \leq \mu(\sigma(\mathcal{A})) \). This implies that \( \sigma(\mathcal{A}) \subseteq \mu^{-1}([t, 1]) \). If \( A \subseteq \mu^{-1}([t, 1]) \), then \( \mu(A) = s, s \geq t \). Let \( \mathcal{A} = \{ B \in \mu^{-1}([t, 1]) : B \subseteq A \} \). For each \( x \in A \), since the unique arc \( \mathcal{A} \) in \( C(X) \) from \( \{ x \} \) to \( X \) must meet \( \mu^{-1}(t) \), there is an element \( B \in \mu^{-1}(t) \) such that \( x \in B \) and \( B \subseteq A \) [KE]. We would like to show that \( \mathcal{A} \subseteq C(\mu^{-1}(t)) \). Consider the projection mapping \( f \) of \( X \) onto the space \( \mu^{-1}(t) \) defined by \( f(x) = B \) if \( x \in B \). This function is continuous [R], and \( f(A) = \mathcal{A} \). Since \( A \) is a subcontinuum of \( X \), so is \( \mathcal{A} \) in \( \mu^{-1}(t) \). Therefore \( \mathcal{A} \subseteq C(\mu^{-1}(t)) \). Thus \( \sigma(\mathcal{A}) = A \) and \( \sigma \) is a continuous mapping of \( C(\mu^{-1}(t)) \) onto \( \mu^{-1}([t, 1]) \). The fact that \( \sigma \) is one-to-one follows from (b) above. Therefore \( \sigma : C(\mu^{-1}(t)) \to \mu^{-1}([t, 1]) \) is a homeomorphism.
We let \( h_t : C(X) \to \mu^{-1}[t, 1] \) be the homeomorphism defined by \( h_t = \sigma^{k-1} h_1 \). The lemma is now proved.

**Lemma 2.2.** Suppose \( X \) is a pseudoarc and \( 0 \leq t < 1 \). Then there is a mapping \( g_t : C(X) \to C(X) \) such that \( g_t \) restricted to \( \mu^{-1}[t, 1] \) is \( h_t^{-1} \) and \( g_t[\mu^{-1}[0, t]] = \mu^{-1}(0) \).

**Proof.** For each \( A \in \mu^{-1}[0, t] \) there is a unique set \( B \subseteq \mu^{-1}(A) \) such that \( A \subseteq B \). Let \( g_t(A) = h_t^{-1}(B) \). It is clear that \( g_t \) is continuous on \( \mu^{-1}[0, t] \). If \( g_t \) is defined to be \( h_t^{-1} \) on \( \mu^{-1}[t, 1] \) then the desired mapping is constructed.

Since \( \mu \) is a closed continuous mapping, we have immediately the following lemma.

**Lemma 2.3.** For each closed set \( F \) and open set \( \emptyset \subseteq \mu^{-1}[F] \), there is an open set \( Q \) such that \( \mu^{-1}[F] \subseteq \mu^{-1}[Q] \subseteq \emptyset \).

Finally, we remark that, if \( h : C(X) \to C(X) \) is a homeomorphism and \( X \) is a pseudoarc then necessarily \( h[\mu^{-1}(0)] = \mu^{-1}(0) \) and \( h(X) = X \).

3. The dimension of the hyperspace of a pseudoarc. In this section we prove two theorems concerning the dimension of the hyperspace \( C(X) \) of a pseudoarc \( X \). The first theorem has been established by Eberhart and Nadler [EN]. The present proof is new and relies only on properties of the pseudoarc. In the above-mentioned paper, it is observed that \( C(X) \) is of dimension two at each point of \( \mu^{-1}(0, 1) = \{ A \in C(X) : 0 < \mu(A) < 1 \} \).

The second theorem of this section shows that \( C(X) \) is also of dimension two at \( X \). This fact will be used in the next section to prove the main theorem.

**Theorem 3.1.** If \( X \) is a pseudoarc then the dimension of \( C(X) \) is two.

**Proof.** Since \( C(X) \) is a nondegenerate continuum, we have \( \dim C(X) \geq 1 \). From Theorem VI.7 of [HW], we have

\[
\dim C(X) \leq \dim \mu[C(X)] + \sup \{ \dim \mu^{-1}(t) : 0 \leq t \leq 1 \}.
\]

From §2 above, we have that the right side of the above inequality is two since the dimension of a pseudoarc is one. We need to prove \( \dim C(X) \neq 1 \).

Suppose \( \dim C(X) = 1 \). Since \( C(X) \) is contractible [R], any mapping on a subcontinuum of \( C(X) \) into \( S^1 \) is inessential, and therefore each subcontinuum of \( C(X) \) has property (b). Thus, each subcontinuum of \( C(X) \) is unicoherent [WH, p. 226]. But there are subcontinua of \( C(X) \) which are not unicoherent. For example, \( \mu^{-1}(0) \cup \mathcal{A}_{x,y} \), where \( \mathcal{A}_{x,y} \) is the unique arc in \( C(X) \) between \( \{ x \} \) and \( \{ y \} \), \( x \neq y \), \( x, y \in X \). Since a pseudoarc contains no arc, \( \mu^{-1}(0) \cap \mathcal{A}_{x,y} = \{ x, y \} \). Consequently, \( \dim C(X) \neq 1 \) and the theorem is proved.
THEOREM 3.2. If $X$ is a pseudoarc then $C(X)$ has dimension two at the point $X$.

PROOF. The proof is by contradiction. Since $C(X)$ is a nondegenerate continuum, we have that the dimension of $C(X)$ at $X$ is no less than one. We will show that the assumption that $C(X)$ is of dimension one at the point $X$ implies $\dim C(X)=1$. The proof will be made in three parts.

Part 1. Let $0<t<1$. If $C(X)$ has dimension one at $X$ then there are two disjoint open sets $\mathcal{S}$ and $\mathcal{T}$ of $C(X)$ such that $\mu^{-1}(0) \subset \mathcal{S}$, $\mu^{-1}[t, 1] \subset \mathcal{T}$, and their boundaries have $\dim \text{Bd}(\mathcal{S})=0=\dim \text{Bd}(\mathcal{T})$.

Let $O$ be an open neighborhood of $X$ such that $O \subset \mu^{-1}[\frac{1}{2}, 1]$ and $\dim \text{Bd}(O)=0$. Then, if $\mathcal{S}$ is the complement of the closure of $O$, $\mathcal{S}$ is an open set containing $\mu^{-1}(0)$ and $\dim \text{Bd}(\mathcal{S})=0$.

Let $P_0 \in O$ and $P_0 \neq X$. As Eberhart and Nadler in [EN] observed, Theorem 15 of [Bl] implies for each $A \in \mu^{-1}[t, 1]$, $A \neq X$, there exists a homeomorphism $h_A:C(X) \to C(X)$ such that $h_A(P_0)=A$. Associated with this homeomorphism are two disjoint open sets $O_A$ and $P_A$ for which $A \in O_A$, $\mu^{-1}(0) \subset P_A$, and $\dim \text{Bd}(O_A)=0=\dim \text{Bd}(P_A)$. Now, $\{O_A:A \in \mu^{-1}[t, 1], A \neq X\}$ is an open cover of the compact set $\mu^{-1}[t, 1]$. Let $O_{A_1}, \ldots, O_{A_n}$ be a subcover, $\mathcal{T}=\bigcup_{i=1}^{n} O_{A_i}$ and $\mathcal{S}=\bigcap_{i=1}^{n} P_{A_i}$. Then $\mathcal{S}$ and $\mathcal{T}$ are disjoint open sets with $\mu^{-1}(0) \subset \mathcal{S}$ and $\mu^{-1}[t, 1] \subset \mathcal{T}$. Since $\text{Bd}(\mathcal{S}) \subset \bigcup_{i=1}^{n} \text{Bd}(P_{A_i})$ and $\text{Bd}(\mathcal{T}) \subset \bigcup_{i=1}^{n} \text{Bd}(O_{A_i})$, we have $\dim \text{Bd}(\mathcal{S})=0=\dim \text{Bd}(\mathcal{T})$ and the first part is proved.

Part 2. Let $0 \leq t \leq 1$ and $O$ be an open neighborhood of $\mu^{-1}(t)$. Suppose the conclusion of Part 1 holds. Then there is an open neighborhood $\mathcal{W}$ of $\mu^{-1}(t)$ such that $\mathcal{W} \subset O$ and $\dim \text{Bd}(\mathcal{W})=0$.

PROOF. By Lemma 2.3, there are two numbers $s_1$ and $s_2$ such that $s_1 < t < s_2$ and $\mu^{-1}[s_1, s_2] \subset O$. We assume for convenience that $0 < t < 1$. The contrary cases involve only a slight modification of the argument. We may now further assume $0 \leq s_1 < t < s_2 \leq 1$.

Let us consider $s_1$. By Lemma 2.2 there is a mapping $g_{s_1}:C(X) \to C(X)$ such that $g_{s_1}$ maps $\mu^{-1}[s_1, 1]$ homeomorphically onto $C(X)$ and $g_{s_1}[\mu^{-1}(0), s_1]=\mu^{-1}(0)$. Hence by Lemma 2.3 there is a number $T_1$ with $0 < T_1$ such that $\mu^{-1}[0, T_1] \cap g_{s_1}[\mu^{-1}[t, 1]]=\emptyset$. From Part 1 there is an open set $\mathcal{T}$ such that the closure of $\mathcal{T}$ does not meet $\mu^{-1}(0), \mathcal{T} \supset \mu^{-1}[T_1, 1]$ and $\dim \text{Bd}(\mathcal{T})=0$. Thus, if $\mathcal{W}=g_{s_1}(\mathcal{T})$ then $\mathcal{W}$ is open, $\mu^{-1}(t) \subset \mathcal{W}$, and $\dim \text{Bd}(\mathcal{W})=0$.

Next, consider $t$. By Lemma 2.2 there is a mapping $g_t:C(X) \to C(X)$ such that $g_t$ maps $\mu^{-1}[t, 1]$ homeomorphically onto $C(X)$ and $g_t[\mu^{-1}[0, t]]=\mu^{-1}(0)$. Hence by Lemma 2.3 there is a number $T_2$ with $0 < T_2$ such that $g_t[\mu^{-1}[T_2, 1]]=\mu^{-1}(0)$ and $\dim \text{Bd}(\mathcal{W})=0$.
\[ \mu^{-1}[0, T_2] \cap g_{r}([\mu^{-1}[s_2, 1]] = \emptyset. \] From Part 1, there is an open set \( \mathcal{S} \) such that the closure of \( \mathcal{S} \) does not meet \( \mu^{-1}[T_2, 1] \), \( \mathcal{S} \cap \mu^{-1}(0) = \emptyset \). Thus, if \( \mathcal{W}_2 = g_{t}^{-1}(\mathcal{S}) \) then \( \mathcal{W}_2 \) is open, \( \mu^{-1}(t) \subset \mathcal{W}_2 \subset \mu^{-1}[0, s_2] \) and \( \dim \text{Bd}(\mathcal{W}_2) = 0. \)

Let \( \mathcal{W} = \mathcal{W}_1 \cap \mathcal{W}_2. \) Then \( \mathcal{W} \) is open, \( \mu^{-1}(t) \subset \mathcal{W} \subset \mu^{-1}[s_1, s_2] \subset \emptyset \) and \( \dim \text{Bd}(\mathcal{W}) \leq \dim \text{Bd}(\mathcal{W}_1) + \dim \text{Bd}(\mathcal{W}_2) = 0. \) Thus Part 2 is proved.

**Part 3.** If \( C(X) \) has dimension one at \( X \) then \( \dim C(X) = 1. \)

**Proof.** Let \( \mathcal{N} = \{\mu^{-1}(t) : 0 \leq t \leq 1\} \). Then \( \mathcal{N} \) is a family of closed subsets of \( C(X) \). By Part 2, each neighborhood of \( \mu^{-1}(t) \) contains a neighborhood whose boundary has dimension zero. Since \( \dim \mu^{-1}(t) \leq 1 \) for each \( t \), we have, by Proposition 8 on p. 90 of [HW], \( \dim C(X) = \dim \bigcup \mathcal{N} \leq 1 \), a contradiction to Theorem 3.1. Thus Theorem 3.2 is proved.

**4. Proof of the main theorem.** We are now in a position to prove our main theorem. Lemma 2.3 provides us with the fact that the family \( \mu^{-1}[r, 1] \), \( 0 \leq r < 1 \), forms a basis of closed neighborhoods of the point \( X \) in \( C(X) \). We infer from Lemma 2.1 that we need only consider the neighborhood \( C(X) \).

**Theorem 4.1.** If \( X \) is a pseudoarc then \( C(X) \) is a two-dimensional Cantor manifold.

**Proof.** By denying the conclusion, we will establish a contradiction to Theorem 3.2. That is, we will show that the existence of a zero-dimensional separator of \( C(X) \) implies the existence of an open neighborhood of \( X \), disjoint with \( \mu^{-1}(0) \), whose boundary has dimension zero. Then the preliminary remarks of this section will complete the proof.

Suppose \( \mathcal{P} \) is a closed zero-dimensional subset of \( C(X) \) which separates \( C(X) \). Let \( \mathcal{A} \) and \( \mathcal{B} \) be nonvoid open sets such that \( C(X) - \mathcal{P} = \mathcal{A} \cup \mathcal{B} \). We will consider two cases.

**Case 1.** Suppose \( X \notin \mathcal{P} \). Without loss of generality, we may assume \( X \in \mathcal{A} \). There are now two possibilities. Either \( \mathcal{P} \cap \mu^{-1}(0) = \emptyset \) or \( \mathcal{P} \cap \mu^{-1}(0) \neq \emptyset \). Let us dispose of the first possibility.

(a) *Suppose \( \mathcal{P} \cap \mu^{-1}(0) = \emptyset. \) In the event that \( \mu^{-1}(0) \subset \mathcal{B} \), the desired neighborhood of \( X \) is \( \mathcal{A} \) and the contradiction is established. Since \( \mu^{-1}(0) \) is connected, \( \mu^{-1}(0) \notin \mathcal{B} \) implies \( \mu^{-1}(0) \subset \mathcal{A} \). \( \mathcal{B} \) being nonvoid, choose \( P \in \mathcal{B} \). \( P \) is a nondegenerate subcontinuum of \( X \) since \( P \notin \mu^{-1}(0) \). Hence \( P \) is a pseudoarc. \( C(P) \) is homeomorphic to \( C(X) \) and \( C(P) \) is a subspace of \( C(X) \). Clearly, \( \mathcal{B} \cap C(P) \) is an open neighborhood of \( P \) in \( C(P) \), disjoint with \( C(P) \cap \mu^{-1}(0) = \{\{p\}: p \in P\} \), whose boundary in \( C(P) \) has dimension zero. Hence, the required neighborhood of \( X \) exists and the
contradiction is established. Thus, we have disposed of the possibility $\mathcal{S} \cap \mu^{-1}(0) = \emptyset$.

(b) Suppose $\mathcal{S} \cap \mu^{-1}(0) \neq \emptyset$. Either $B \cap \mu^{-1}(0) \neq \emptyset$ or $B \cap \mu^{-1}(0) = \emptyset$. Suppose first that $B \cap \mu^{-1}(0) \neq \emptyset$. Let $P_0 \in B \cap \mu^{-1}(0)$ and $A \in \mu^{-1}(0)$. Then, both $P_0$ and $A$ are singleton subsets of $X$. Since $X$ is homogeneous, there is a homeomorphism $h_A: C(X) \to C(X)$ such that $h_A(P_0) = A$. Associated with each such homeomorphism are two disjoint open sets $\mathcal{O}_A = h_A(\mathcal{A})$ and $\mathcal{P}_A = h_A(\mathcal{B})$ with the properties $X \in \mathcal{O}_A$ and $\dim \partial(\mathcal{O}_A) = 0$. Since $\{\mathcal{P}_A: A \in \mu^{-1}(0)\}$ is an open cover of the compact set $\mu^{-1}(0)$, there is a finite cover $\mathcal{P}_{A_1}, \ldots, \mathcal{P}_{A_n}$. Let $\mathcal{O} = \bigcap_{i=1}^n \mathcal{O}_{A_i}$ and $\mathcal{P} = \bigcup_{i=1}^n \mathcal{P}_{A_i}$. Then $\mathcal{O}$ and $\mathcal{P}$ are disjoint open sets, $X \in \mathcal{O}$, $\mu^{-1}(0) \subset \mathcal{P}$ and $\dim \partial(\mathcal{O}) = 0$. Thus, the desired neighborhood of $X$ is found and the contradiction established.

Next, suppose $B \cap \mu^{-1}(0) = \emptyset$. Since $X \neq B$ and $B \neq \emptyset$, there is a non-degenerate subcontinuum $P \in B$. $P$ is a pseudoarc and $C(P)$ is a subspace of $C(X)$ which is homeomorphic to $C(X)$. Since $\dim [C(P) \cap \mu^{-1}(0)] = 1$ and $\dim \mathcal{S} = 0$, we have $[C(P) \cap \mu^{-1}(0)] - \mathcal{S}$ is a nonempty subset of $\mathcal{A} \cap C(P)$. By considering $C(P)$, $\mathcal{S}' = C(P) \cap \mathcal{S}$, $\mathcal{A}' = B \cap C(P)$ and $\mathcal{B}' = \mathcal{A} \cap C(P)$, we see that $\mathcal{S}'$ is a zero-dimensional separator of $C(P)$, $C(P) - \mathcal{S}' = \mathcal{A'} \cup \mathcal{B}'$ where $\mathcal{A}'$ and $\mathcal{B}'$ are open sets, $P \in \mathcal{A}'$ and $\mathcal{B}' \cap \mu^{-1}(0) \neq \emptyset$ where $\mu_P$ is a $\mu$ function associated with the pseudoarc $P$. We have arrived at the situation which immediately preceded the one at hand.

Now the two possibilities (a) and (b) under Case 1 have been completely disposed of.

Case 2. Suppose $X \in \mathcal{S}$. We will dispose of this case by reducing it to Case 1.

For each $x \in X$, there is a unique arc $\mathcal{A}_x$ in $C(X)$ from $\{x\}$ to $X \setminus \{x\}$. Let $M = \{x \in X: \mathcal{A}_x \cap \mathcal{A} \neq \emptyset\}$ and $N = \{x \in X: \mathcal{A}_x \cap \mathcal{B} \neq \emptyset\}$. Since $\dim \mathcal{S} = 0$, we have $\emptyset \neq \mathcal{A}_x - \mathcal{S} \subset \mathcal{A} \cup \mathcal{B}$ for each $x \in X$. Consequently, $X = M \cup N$. We will show $M \neq \emptyset$ and open. A symmetric argument shows $N \neq \emptyset$ and open. To this end, we recall a continuous mapping $\Phi: X \times [0, 1] \to C(X)$ defined in Theorem 3.5 of [R]. $\Phi$ is defined as

$$\Phi(x, t) = A,$$

where $x \in A \in C(X)$ and $\mu(A) = t$.

Since each pair of points in $C(X)$ has a unique arc between them, we have $\mathcal{A}_x = \emptyset([x] \times [0, 1])$. Consequently, $M = F[\Phi^{-1}(\mathcal{A})]$, where $F$ is the natural projection $F: X \times [0, 1] \to X$.

Since $X$ is connected $M \cap N \neq \emptyset$. Let $x \in M \cap N$ and $P \in \mathcal{A}_x \cap \mathcal{A}$ and $Q \in \mathcal{A}_x \cap \mathcal{B}$. Since $P$ and $Q$ are in the arc $\mathcal{A}_x$, either $P \supseteq Q$ or $P \subseteq Q$. Also, $P \neq Q$. Suppose $P \supseteq Q$. By considering the pseudoarc $P$, we have for $C(P)$, $\mathcal{S}' = C(P) \cap \mathcal{S}$, $\mathcal{A}' = \mathcal{A} \cap C(P)$ and $\mathcal{B}' = B \cap C(P)$, precisely the Case 1. Similar considerations apply when $P \subseteq Q$.

The main theorem is now established.
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