PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 31, No. 2, February 1972

VANISHING OF STIEFEL-WHITNEY CLASSES

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Atiyah and Hirzebruch proved in [**1**] that for any vector bundle ξ over the 9-fold iterated suspension of a finite CW-complex, the mod 2 Stiefel-Whitney class $w_i(\xi)$ is zero for all $i>0$. A complement to this for 2-, 3-, and 5-fold suspensions is included in the following:

REMARK. *Let ξ be a spherical fibring over the (2^k+1)-fold suspension of a* CW-*complex (k any nonnegative integer). Then $w_i(\xi)=0$ for $i\not\equiv 0 \bmod 2^{k+1}$.*

The Hopf bundles over spheres show that, when $k\leqq 2$, this extent of vanishing is best possible for the given number of suspensions, even when we restrict to sphere bundles. By [**1**], it is not best possible for sphere *bundles* when $k\geqq 3$; but an example in §4 of [**2**] shows that it is best possible for spherical *fibrings* when $k=3$. In fact that example generalises to any k for which the Whitehead product $[\iota, \iota]$ can be halved, where ι generates $\pi_n(S^n)$ and $n=2^{k+1}-1$.

The remark follows from the Wu formulae (see [**3**]). For given $i\not\equiv 0 \bmod 2^{k+1}$, we may write $i=2^{r+1}\cdot m+2^r$, where m, r are nonnegative integers and $r\leqq k$. Now put $s=2^r\cdot m$, $t=2^r(m+1)$, and consider the Wu formula for $Sq^s w_t$. Using the vanishing of cup-products on a suspension, this yields $Sq^s w_t=w_i$. But $Sq^s w_t=0$ since we are on a $(t-s+1)$-fold suspension.

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Received by the editors July 8, 1971.

AMS 1970 *subject classifications*. Primary 55F40, 55D40.

Key words and phrases. Stiefel-Whitney classes, suspension.