EQUIVALENCE OF ALGEBRA NORMS ON $C(G)$

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In [1] Slobko shows that if $G$ is a compact first countable group, if $S_a$ denotes the translation operator on the algebra of continuous functions on $G$ (with pointwise multiplication),

$$S_a f(x) = f(ax), \quad f \in C(G),$$

and if $| \cdot |$ is an algebra norm on $C(G)$ with respect to which the mapping $a \rightarrow S_a f$ is continuous for each $f$ in $C(G)$ then $| \cdot |$ and the sup norm $\| \cdot \|$ are equivalent. The following simple argument shows that the countability assumption is redundant.

Define

$$\|f\| = \sup_{a \in G} |S_a f|, \quad f \in C(G).$$

Then since $G$ is compact this defines an algebra norm on $C(G)$ and since

$$\|S_a\| = \sup_{\|f\| \leq 1} \|S_a f\| = \sup_{\|f\| \leq 1} \sup_{b \in G} |S_b S_a f|$$

$$= \sup_{\|f\| \leq 1} \sup_{a \in G} |S_a f| = 1,$$

each mapping $S_a : C(G) \rightarrow C(G)$ is continuous with respect to $\|\cdot\|$. Hence by Theorem 1 of [1], $| \cdot |$ and $\| \cdot \|$ are equivalent. But clearly $\|f\| \geq |f|$, so $| \cdot |$ and $\| \cdot \|$ are equivalent.

REFERENCE


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