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SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

PROJECTIONS OF BANACH SPACES

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We deduce Phillips' projection theorem ([2], [3]) from a lemma on weak convergence. Say that a normed space E has property WS if every weak* $(=\sigma(E', E))$ null sequence is weak $(=\sigma(E', E''))$ null. It is known ([1], [2], [3]) that m (the bounded complex sequences) has WS.

LEMMA. *The property* WS *is preserved under bounded projections.*

Let $P: X \to E$ be a bounded projection of X onto its subspace E, and $\{v_n\}$ a weak* null sequence in E'. Let $u_n = P'(v_n) \in X'$. Since P' is an adjoint map, $\{u_n\}$ is weak* null, hence weak null, by hypothesis. The map $u \to u|E$ is a bounded projection of X' onto E' carrying u_n to v_n. Thus $\{v_n\}$ is weak null.

COROLLARY. *There exists no bounded projection of m onto c_0.*

For we take $X = m$, $E = c_0$ in the Lemma.

REMARKS. 1. The method of [4] is to find a hereditary property of m which m/c_0 does not have. We found a nonhereditrary property preserved under projection.

2. An easy extension is available. All we used about the projection is that it is bounded and has a bounded right inverse Q. (Use Q' instead of $u \to u|E$ in the proof.) Hence we see that there is no bounded map of m onto c_0 which has a right inverse.

3. If E is isomorphic to a dual space there is always a bounded projection of E'' onto E. Thus the Lemma yields the known fact that c_0 is not isomorphic to a dual space. (Take $X = c_0'' = m$.)

4. Remark 3 also shows that if $E^{(n)}$ has WS so does $E^{(n-2)}$ for all $n > 2$.

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