PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 36, Number 1, November 1972

NONRECURSIVE RELATIONS AMONG THE ISOLS

ERIK ELLENTUCK[1]

ABSTRACT. The universal isol metatheorem is extended so as to deal with nonrecursive relations and countable Boolean operations.

1. **Introduction.** Let $\mathscr{L}$ be a first order language with equality containing an infinite list of individual variables $v_0, v_1, \cdots$, and for each $n<\omega$ an individual n, an n-ary function symbol f for each almost recursive combinatorial $f: X^n\omega \to \omega$, and an n-ary relation symbol R for each relation $R \subseteq X^n\omega$. Terms are built up from variables, constants, and function symbols by composition, and atomic formulas are of the form $\tau_0=\tau_1$ or $\mathrm{R}(\tau_0, \cdots, \tau_{n-1})$ where $\tau_0, \cdots, \tau_{n-1}$ are terms and R is an n-ary relation symbol. Formulas and sentences are defined as usual. $\mathscr{L}$ has the standard interpretation in ω (write $\omega \models \mathfrak{A}[x]$ for "the assignment x satisfies $\mathfrak{A}$ in ω") and is interpreted in Λ by letting f and R denote f_Λ and R_Λ respectively (write $\Lambda \models \mathfrak{A}[x]$ for "the assignment x satisfies $\mathfrak{A}$ in Λ"). Throughout this paper $\mathfrak{A}$ will denote a quantifier-free conjunctive normal form formula all of whose free variables are among $v_0, \cdots, v_{k-1}$ and all of whose relation symbols occurring negated in some conjunct of $\mathfrak{A}$ are among $\mathrm{R}_0, \cdots, \mathrm{R}_{n-1}$. Whenever we wish to stress these symbols we write $\mathfrak{A}(R_0, \cdots, R_{n-1})$ and let $\mathfrak{A}(R'_0, \cdots, R'_{n-1})$ be the result of replacing each negated occurrence of R_i in $\mathfrak{A}$ by R'_i for $i<n$ (provided each R_i and R'_i have the same ary-ness). We assume that the reader is familiar with the notions Λ, Λ^∞, totally unbounded, specification (S_h), and Horn reduction. A set $R \subseteq X^k\omega$ is *eventual* if its complement $X^k\omega - R$ is not totally unbounded. It will also be convenient to introduce an improper notion $\Lambda^\infty \models (\forall v_0, \cdots, v_{k-1})\mathfrak{A}$ to mean $\Lambda \models \mathfrak{A}[x]$ for every $x \in X^k\Lambda^\infty$. Our starting point is the fundamental metatheorem of [3] which characterizes universal sentences in Λ.

THEOREM 11.1 OF [3]. *If all relation symbols occurring in $\mathfrak{A}$ denote recursive relations then*

(i) $\Lambda^\infty \models (\forall v_0, \cdots, v_{k-1})\mathfrak{A}$ *if and only if there is a Horn reduction $\mathfrak{A}'$ of $\mathfrak{A}$ such that $\{x \in X^k\omega : \omega \models \mathfrak{A}'[x]\}$ is eventual.*

Received by the editors April 25, 1971.

AMS 1970 *subject classifications*. Primary 02F40.

Key words and phrases. Isols, universal sentence, Horn reduction.

[1] Prepared while the author was partially supported by National Science Foundation grant GP-11509.

(ii) $\Lambda \models (\forall v_0, \cdots, v_{k-1})\mathfrak{A}$ *if and only if* $\omega \models (\forall v_0, \cdots, v_{k-1})\mathfrak{A}$, *and for each* $\sigma \subsetneq k$ *and* $h: \sigma \to \omega$ *there is a Horn reduction* $\mathfrak{A}'$ *of* $\mathfrak{A}$ *such that* $S_h\{x \in \times^k\omega : \omega \models \mathfrak{A}'[x]\}$ *is eventual.*

In our first result we remove the restriction that the relation symbols in $\mathfrak{A}$ denote recursive relations. Thus we have

THEOREM 1. (i) $\Lambda^\infty \models (\forall v_0, \cdots, v_{k-1})\mathfrak{A}(R_0, \cdots, R_{n-1})$ *if and only if for each sequence of recursively enumerable relations* $R'_0, \cdots, R'_{n-1}$ *with* $R'_i \subseteq R_i$ *for* $i<n$, *there is a Horn reduction* $\mathfrak{A}'$ *of* $\mathfrak{A}$ *such that*

$$\{x \in \times^k\omega : \omega \models \mathfrak{A}'(R'_0, \cdots, R'_{n-1})[x]\} \text{ is eventual.}$$

(ii) $\Lambda \models (\forall v_0, \cdots, v_{k-1})\mathfrak{A}(R_0, \cdots, R_{n-1})$ *if and only if*

$$\omega \models (\forall v_0, \cdots, v_{k-1})\mathfrak{A}(R_0, \cdots, R_{n-1})$$

and for each sequence of recursively enumerable relations $R'_0, \cdots, R'_{n-1}$ *with* $R'_i \subseteq R_i$ *for* $i<n$, *and for each* $\sigma \subsetneq k$ *and* $h: \sigma \to \omega$ *there is a Horn reduction* $\mathfrak{A}'$ *of* $\mathfrak{A}$ *such that* $S_h\{x \in \times^k\omega : \omega \models \mathfrak{A}'(R'_0, \cdots, R'_{n-1})[x]\}$ *is eventual. If all* R_i *are recursively enumerable suppress all mention of* R'_i *above.*

Note that the replacements made above of R'_i for R_i are only at the negated occurrences of R_i in $\mathfrak{A}$. Amusing consequences of Theorem 1 are: (i) If $R \subseteq \omega$ is immune with immune complement $S = \omega - R$ then $\Lambda^\infty \models (\forall v_0)(R(v_0) \to S(v_0))$ by Theorem 1(i) although nothing could be falser in ω. (ii) If $R \subseteq \omega$ is immune and is expressed as the union $R = S_0 \cup S_1$ of two infinite disjoint sets then $\Lambda \models (\forall v_0)(R(v_0) \to S_0(v_0) \vee S_1(v_0))$ by Theorem 1 (ii) even though neither Horn reduction is eventual in ω. Note, however, that these examples readily follow from the fact that $R_\Lambda = R$ for immune R (cf. Theorem 4.1 of [3]). These examples in no way illustrate the strength of Theorem 1 which is concerned rather with getting results when function symbols are present. It would be desirable of course to develop a theory which would allow for not necessarily recursive almost combinatorial functions; however, the well-known result that in general composition of such functions does not commute with their extension makes us pessimistic of such a possibility.

Our second result concerns a generalization of Theorem 1 to a class of infinitary universal sentences in $\mathscr{L}_{\omega_1\omega_1}$. The basic symbols of this language will be the same as those of $\mathscr{L}$ except now we allow countable conjunctions, disjunctions, and application of countable homogeneous quantifier blocks. In this paper $\mathfrak{B}$ will denote a quantifier free formula of $\mathscr{L}_{\omega_1\omega_1}$ consisting of a countable conjunction of countable disjunctions of atomic formulae and their negations. We assume that the free variables of $\mathfrak{B}$ are among $v_0, v_1, \cdots$, and that the relation symbols occurring negated in some conjunct of $\mathfrak{B}$ are among $R_0, R_1, \cdots$. We stress these symbols by writing $\mathfrak{B}(R_0, R_1, \cdots)$ and let $\mathfrak{B}(R_0', R_1', \cdots)$ be the result of replacing each negated occurrence of R_i in $\mathfrak{B}$ by R_i' for $i<\omega$ (provided each R_i and R_i' have the same ary-ness). Let $v=\langle v_0, v_1, \cdots\rangle$ and express the universal closure of $\mathfrak{B}$ as $(\forall v)\mathfrak{B}$. Let $\mathfrak{B}'$ be any conjunct of $\mathfrak{B}$. We say that $\mathfrak{B}''$ is a *finite approximation* of $\mathfrak{B}'$ if $\mathfrak{B}''$ is obtained from $\mathfrak{B}'$ by striking out all but a finite positive number of disjuncts of $\mathfrak{B}'$. Thus $\mathfrak{B}''$ is an ordinary formula of $\mathscr{L}$ whose universal quantification can be expressed as $(\forall)v\mathfrak{B}''$.

THEOREM 2. $\Lambda^\infty \models (\forall v)\mathfrak{B}(R_0, R_1, \cdots)$ *if and only if for each sequence of recursively enumerable relations* $R_0'', R_1'', \cdots$, *with* $R_i'' \subseteq R_i$ *for* $i<\omega$, *and for each conjunct* $\mathfrak{B}'$ *of* $\mathfrak{B}$, *there is a finite approximation* $\mathfrak{B}''$ *of* $\mathfrak{B}'$ *such that* $\Lambda^\infty \models (\forall v)\mathfrak{B}''(R_0'', R_1'', \cdots)$. *If all* R_i *are recursively enumerable suppress all mention of* R_i'' *above.*

Amusing consequences of Theorem 2 are: (i) If $R_i=\omega-\{i\}$ and $\mathfrak{B}$ is $(R_1(v_0)\wedge R_2(v_0)\wedge\cdots)\rightarrow v_0=0)$ then $(\forall v)\mathfrak{B}$ is an infinitary universal Horn sentence true in ω but false in Λ since no finite approximation of $\mathfrak{B}$ holds eventually in ω. (ii) If $R\subseteq\omega$ is a recursively enumerable nonrecursive set and $S_0, S_1, \cdots$ is an enumeration of all the recursive subsets of R, and $\mathfrak{B}$ is $R(v_0)\rightarrow(S_0(v_0)\vee S_1(v_0)\vee\cdots)$, then $(\forall v)\mathfrak{B}$ is true in ω but false in Λ since no finite approximation of $\mathfrak{B}$ holds eventually in ω. Example (ii) is Corollary 3.9 of [**4**]. It is an interesting result because it implies that the method of extending relations by frames is indeed stronger than the method of extending relations by equations.

2. **Proofs.** The proof of Theorem 1 depends on a simple lemma which was probably overlooked because no one was looking for it. We have

LEMMA 1. *If* $R\subseteq \times^n\omega$, $S\subseteq\times^k\omega$, *and* $f_i:\times^k\omega\rightarrow\omega$, *for* $i<n$, *are recursive combinatorial functions such that* $(\forall x\in\times^k\omega)$ $(x\in S$ *iff* $\langle f_0(x), \cdots, f_{n-1}(x)\rangle\in R)$ *then for all* $x\in\times^k\Lambda$, $x\in S_\Lambda$ *if and only if* $\langle f_{0\Lambda}(x), \cdots, f_{(n-1)\Lambda}(x)\rangle\in R_\Lambda$.

PROOF. We extend ordinary set theoretic notation componentwise to k-tuples as follows. Let $\mathscr{P}$ be the power set operation. If $\alpha, \beta\in\times^k\mathscr{P}(\omega)$ let $\alpha\subseteq\beta$ if $\alpha_i\subseteq\beta_i$ for $i<k$, let $\alpha\cap\beta$ be that element of $\times^k\mathscr{P}(\omega)$ such that $(\alpha\cap\beta)_i=\alpha_i\cap\beta_i$ for $i<k$ and let $|\alpha|$ be that sequence of cardinals such that $|\alpha|_i=|\alpha_i|$ for $i<k$. If $A\subseteq\times^k P(\omega)$ let $\bigcup A\in\times^k P(\omega)$ satisfy $(\bigcup A)_i=\bigcup\{\alpha_i:\alpha\in A\}$ for $i<k$. Finally let Q be the set of all finite subsets of ω. Now for our proof. Let $x=\langle x_0, \cdots, x_{k-1}\rangle\in S_\Lambda$ and choose $\xi=\langle\xi_0, \cdots, \xi_{k-1}\rangle$ such that $x_i=\mathrm{Req}(\xi_i)$ for $i<k$. Let G be a recursive

S-frame such that ξ is attainable from G, in symbols $\xi \in \mathscr{A}(G)$ and let φ_i, $i<n$, be recursive combinatorial operators inducing f_i. Write $\varphi(\alpha)$ for $\langle\varphi_0(\alpha), \cdots, \varphi_{n-1}(\alpha)\rangle$. One direction of our lemma follows by showing that $F=\{\varphi(\alpha):\alpha \in G\}$ is a recursive R-frame for which $\varphi(\xi) \in \mathscr{A}(F)$. Now F^* is recursively enumerable because $F^*=\{\beta:(\exists\alpha \in G^*)(\beta\subseteq \varphi(\alpha))\}$. If $\gamma \in F^*$ then $\gamma\subseteq \varphi(\alpha)$ for some $\alpha \in G$. Next we define

$$\Sigma(\gamma) = \varphi(C_G(\bigcup \{y:(\exists i < n)(\exists x \in \gamma_i)(y = \varphi_i^{-1}(x))\}))$$

and note that Σ is partial recursive and $\gamma\subseteq \Sigma(\gamma)\subseteq \varphi(\alpha)$. Since $\Sigma(\gamma) \in F$, it is clearly the required C_F. F is closed under $\bigcap$ because G is closed under $\bigcap$ and φ is multiplicative. $\beta \in F$ implies $|\beta| \in R$ follows by hypothesis and the definition of F. Thus F is a recursive R-frame. If $\gamma \in \times^n Q$ and $\gamma\subseteq \varphi(\xi)$ then $\alpha=\bigcup \{y:(\exists i<n)(\exists x \in \gamma_i)(y=\varphi_i^{-1}(x))\}\subseteq \xi$ and hence $\gamma\subseteq \varphi(\alpha)\subseteq \varphi(C_G(\alpha))\subseteq \varphi(\xi)$. But $C_F(\gamma)=\varphi(C_G(\alpha)) \in F$ so that $\varphi(\xi) \in \mathscr{A}(F)$. Conversely suppose that F is a recursive R-frame such that $\varphi(\xi) \in \mathscr{A}(F)$. The other direction of our lemma will follow by showing that $G=\{\alpha:\varphi(\alpha) \in F\}$ is a recursive S-frame for which $\xi \in \mathscr{A}(G)$. Let us first note that $H=\{\varphi(\alpha):\alpha \in \times^k Q\}$ is a recursive frame and that $\varphi(\xi) \in \mathscr{A}(H)$. Since $\varphi(\xi) \in \mathscr{A}(F)$ and $\varphi(\xi)$ is isolated, 3.6 of [3] implies that $F\cap H$ is a recursive R-frame and $\varphi(\xi) \in \mathscr{A}(F\cap H)$. Hence there is no loss of generality by assuming that $F\subseteq H$. Then

$$\alpha = \varphi(\bigcup \{y:(\exists i < n)(\exists x \in \alpha_i)(y = \varphi_i^{-1}(x))\}) \tag{1}$$

for $\alpha \in F$ follows from elementary properties of combinatorial operators. We show that G^* is recursively enumerable, that G is closed under $\bigcap$, and that $\alpha \in G$ implies $|\alpha| \in S$ in exactly the same way as before. If $\gamma \in G^*$ then $\gamma\subseteq \alpha$ for some $\alpha \in G$. Now define

$$\Sigma(\gamma) = \gamma \cup \bigcup \{y:(\exists i < n)(\exists x \in (C_F(\varphi(\gamma)))_i)(y = \varphi_i^{-1}(x))\}$$

and note that Σ is partial recursive and $\gamma\subseteq\Sigma(\gamma)\subseteq \alpha$. Moreover, from (1) we see that $\varphi(\Sigma(\gamma))=C_F(\varphi(\gamma)) \in F$ and consequently $\Sigma(\gamma)$ is the required C_G. Thus G is a recursive S-frame. If $\gamma \in \times^k Q$ and $\gamma\subseteq \xi$ then $\varphi(\gamma)\subseteq C_F(\varphi(\gamma))\subseteq \varphi(\xi)$. It immediately follows from the definition of $\Sigma(\gamma)$ that $\gamma\subseteq\Sigma(\gamma)\subseteq \xi$, i.e., $\xi \in \mathscr{A}(F)$. Q.E.D.

LEMMA 2. *The same as Lemma* 1 *except that we replace the* f_i, $i<n$, *by almost recursive combinatorial functions.*

PROOF. For notational ease assume that R, $S\subseteq \omega$ and f is unary. Our proof below works equally well in the general case. Let us suppose that f is almost recursive combinatorial and $(\forall x \in \omega)(x \in S \text{ iff } f(x) \in R)$. Let $\tilde{R}=\{x \in \times^2\omega:x_0-x_1 \in R\}$ and let f^+, f^- be a pair of unary recursive combinatorial functions such that $(\forall x \in \omega)(f(x)=f^+(x)-f^-(x))$. By definition of $\tilde{R}$, $(\forall x \in \omega)(x \in S$ iff $\langle f^+(x), f^-(x)\rangle \in \tilde{R})$ and hence by Lemma 1 we have $(\forall x \in \Lambda)(x \in S_\Lambda$ iff $\langle f^+_\Lambda(x), f^-_\Lambda(x)\rangle \in \tilde{R}_\Lambda)$. The sentences

$$(\forall v_0, x_1, v_2)(v_0 + v_1 = v_2 \wedge \tilde{\mathrm{R}}(v_2, v_0) \rightarrow \mathrm{R}(v_1)),$$
$$(\forall v_0, v_1, v_2)(v_0 + v_1 = v_2 \wedge \mathrm{R}(v_1) \rightarrow \tilde{\mathrm{R}}(v_2, v_0))$$

are true in ω, have no atomic formula containing both a function symbol and a relation symbol denoting a nonrecursive relation, and hence by Corollary 11.2 of [3], are true in Λ. Thus for all $x_0, x_1 \in \Lambda$ if $x_0 \geqq x_1$ then $\langle x_0, x_1\rangle \in \tilde{R}_\Lambda$ if and only if $x_0 - x_1 \in R_\Lambda$. Since $f_\Lambda(x)=f^+_\Lambda(x)-f^-_\Lambda(x)$ for all $x \in \Lambda$ our lemma follows immediately. Q.E.D.

We also need the following result which is proved in [2].

LEMMA 3. *If* $R \subseteq \times^n\omega$, $S \subseteq \omega$ *are recursively enumerable and* $f_i : \omega \rightarrow \omega$ *for* $i<n$ *are combinatorial functions (which are not necessarily recursive) such that* $(\forall x \in \omega)(x \in S$ *iff* $\langle f_0(x), \cdots, f_{n-1}(x)\rangle \in R)$ *then there is an immune set* θ *such that if* $\xi \subseteq \theta$ *is infinite and* $x = \mathrm{Req}(\xi)$ *then* $x \in S_\Lambda$ *if and only if* $\langle f_{0\Lambda}(x), \cdots, f_{(n-1)\Lambda}(x)\rangle \in R_\Lambda$.

PROOF OF THEOREM 1. (A sketch; refer to [3] for all the details.) As in the proof of Theorem 11.1 of [3] part (ii) reduces to part (i), and for (i) it suffices to consider the case where $\mathfrak{A}$ is a single conjunct. We may also dispense with equality and assume terms have been collapsed to single function symbols. Thus we may assume that $\mathfrak{A}$ has the form

$$\prod_{i<n} \mathrm{R}_i(\mathrm{f}_i(v)) \rightarrow \sum_{i<m} \mathrm{S}_i(\mathrm{g}_i(v)) \tag{2}$$

where $\prod$ ($\sum$) denotes repeated conjunction (disjunction) and if R say, is j-ary, we have written $\mathrm{R}(\mathrm{f}(v))$ as short for

$$\mathrm{R}(\mathrm{f}_0(v_0, \cdots, v_{k-1}), \cdots, \mathrm{f}_{j-1}(v_0, \cdots, v_{k-1})).$$

Now suppose that $\mathfrak{A}$ satisfies the arithmetical condition on ω given in (i) and that $x \in \times^k\Lambda^\infty$ satisfies $\Lambda \models \prod_{i<n} \mathrm{R}_i(\mathrm{f}_i(v))[x]$, i.e., $f_{i\Lambda}(x) \in R_{i\Lambda}$ for $i<n$. By the definition of extension by frames this implies that there are recursively enumerable relations R_i', $R_i' \subseteq R_i$ for $i<n$, such that $f_{i\Lambda}(x) \in R'_{i\Lambda}$, i.e., $\Lambda \models \prod_{i<n} \mathrm{R}_i'(\mathrm{f}_i(v))[x]$. Hence by hypothesis there is a Horn reduction $\mathfrak{A}'$ of $\mathfrak{A}$, such that $\mathfrak{A}'(R_0', \cdots, R_{n-1}')$ has the form

$$\prod_{i<n} \mathrm{R}_i'(\mathrm{f}_i(v)) \rightarrow \mathrm{S}(\mathrm{g}(v)), \tag{3}$$

where $\mathrm{S}(\mathrm{g}(v))$ is one of $\mathrm{S}_i(\mathrm{g}_i(v))$, and which eventually holds in ω. Let $R_i'' = \{x \in \times^k\omega : f_i(x) \in R_i'\}$, $S'' = \{x \in \times^k\omega : g(x) \in S\}$ so that by making the appropriate replacements in (3),

$$\prod_{i<n} \mathrm{R}_i''(v) \to \mathrm{S}''(v) \tag{4}$$

eventually holds in ω. By Corollary 11.2 of [3], (4) will also hold in Λ^∞. By Lemma 2, $x \in R''_{i\Lambda}$ for $i<n$ and hence by (4), $x \in S''_\Lambda$ so that again by Lemma 2 $g_\Lambda(x) \in S_\Lambda$, i.e., $\Lambda \models \mathrm{S}(\mathrm{g}(v))[x]$ and therefore

$$\Lambda \models \sum_{i<m} \mathrm{S}_i(\mathrm{g}_i(v))[x].$$

Thus (2) holds in Λ^∞. Conversely suppose that the arithmetical condition on ω in (i) does not hold, i.e., that there are recursively enumerable R_i', $R_i' \subseteq R_i$ for $i<m$ such that for each $j<m$

$$\prod_{i<n} \mathrm{R}_i'(\mathrm{f}_i(v)) \to \mathrm{S}_j \mathrm{g}_j(v)) \tag{5}$$

does not eventually hold in ω. Define R_i'', S_j'' as above so that

$$\prod_{i<n} \mathrm{R}_i''(v) \to \mathrm{S}_j''(v) \tag{6}$$

does not eventually hold in ω for each $j<m$, and note that the R_i'' are recursively enumerable. Now by the method described in [1] we can construct a k-tuple $h(x)=\langle h_0(x), \cdots, h_{k-1}(x)\rangle$ of not necessarily recursive, but unary combinatorial functions such that for each $i<n$, $j<m$, and $x<\omega$, $h(x) \in R_i''$, but $h(x) \notin S_j''$ for infinitely many values of x. By Lemma 3 there is an immune set $\theta \subseteq \omega$ such that for every infinite subset $\xi \subseteq \theta$, if $x=\mathrm{Req}(\xi)$ then $h_\Lambda(x) \in R''_{i\Lambda}$ for $i<n$ (note that here is the place where we really use the fact that each R_i'' is recursively enumerable). Further by the usual category argument on the Cantor space of subsets of θ we can actually find a $\xi_0 \subseteq \theta$ such that if $x_0=\mathrm{Req}(\xi_0)$ then $u_0=h_\Lambda(x_0) \notin S''_{j\Lambda}$ for $j<m$. We can also guarantee that $u_0 \in \mathsf{X}^k\Lambda^\infty$ by choosing the components of h to be strictly increasing functions. Now use Lemma 2 to show that $f_{i\Lambda}(u_0) \in R'_{i\Lambda} \subseteq R_{i\Lambda}$ for $i<n$ and $g_{j\Lambda}(u_0) \notin S_{j\Lambda}$ for $j<m$. Go back to our formalism and see that u_0 is the required counterexample for (2). Q.E.D.

Having seen the proof of Theorem 1, Theorem 2 requires little argumentation. The positive aspect of the theorem is self-evident; it was the negative side of the theorem which used a counterexample obtained from nonrecursive combinatorial functions which gave us difficulty. In Theorem 1 h was nonrecursive because the S_j were nonrecursive. In Theorem 2 h (an ω-tuple of unary combinatorial functions) is nonrecursive for that reason and also because there need be no effective enumeration of the infinitely many S_j. The details for the construction of h are tedious though straightforward in principle. We use Theorem 1 to show that certain sets are totally unbounded. The interested reader can see this argument in the proof of Theorem 2 of **[2]**. It should be noted that our negative results could also be gotten from a somewhat altered version of the compactness theorem of **[4]**.

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DEPARTMENT OF MATHEMATICS, RUTGERS UNIVERSITY, NEW BRUNSWICK, NEW JERSEY 08903

SCHOOL OF MATHEMATICS, INSTITUTE FOR ADVANCED STUDY, PRINCETON, NEW JERSEY 08540