SHOR TER NOTES

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THE NONEXISTENCE OF COMPLEX HAAR SYSTEMS ON NONPLANAR LOCALLY CONNECTED SPACES

GEORGE W. HENDERSON AND BRIAN R. UMMEL

Let $X$ be a compact Hausdorff space and $C(X)$ be the complex linear space of complex valued continuous functions defined on $X$. An $n$-dimensional subspace $L$ of $C(X)$, $n \geq 2$, is called a complex Haar system on $X$ if every nonzero member of $L$ has at most $n-1$ zeros. The purpose of this note is to prove

**Theorem 1.** If $X$ is locally connected, then a necessary and sufficient condition for the existence of a complex Haar system of $X$ is that $X$ be imbeddable in the plane.

This affirms a conjecture of J. Overdeck and generalizes the theorem of Schoenberg and Yang [3] in which $X$ was assumed to be finite polyhedral. To see how their proof extends to the locally connected case, let $S^2$ be the 2-sphere, $K_5$ and $K_{3,3}$ be the primitive skew curves (i.e. the complete graph on 5 vertices and the houses and wells configuration), and $C_1$ and $C_2$ be the Claytor curves as described on the first page of [1].

**Theorem 2 (Claytor [1]).** If $X$ is a nonplanar Peano continuum, then $X$ contains a subspace homeomorphic to one of $S^2$, $K_5$, $K_{3,3}$, $C_1$, or $C_2$.

Since a complex Haar system on $X$ induces one on each of its subspaces, it suffices to show that none of these five spaces admits a complex Haar system. This was done in [3] for $S^2$, $K_5$, and $K_{3,3}$. Now observe that $C_j$ contains a nonempty open set $U_j$ such that $C_j - U_j$ is homeomorphic to $C_j$, $j=1$ or 2. If there were a complex Haar system on $C_j$, then $C_j - U_j$ would be imbeddable in the plane by Lemma 1 of [2]; but this is impossible since $C_j - U_j$ is homeomorphic to $C_j$. This proves necessity, and the sufficiency is obvious.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WISCONSIN-MILWAUKEE, MILWAUKEE, WISCONSIN 53201

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