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SHORTER NOTES

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# ON THE DIFFERENCE OF TWO SECOND CATEGORY BAIRE SETS IN A TOPOLOGICAL GROUP

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A set in a topological space is called a set with property of Baire (see [2]), or simply a Baire set, if it can be written in the form $G\Delta P$, where $G$ is an open set and $P$ is a set of first category. Recently Kominek proved [1] that the sum and difference (algebraic) of two second category Baire sets in a topological vector space contain nonempty open sets. The purpose of this note is to give a simple proof of this result in a more general setting.

**Theorem.** *Let $G$ be a topological group. If $A, B \subset G$ are second category Baire sets, then both $AB^{-1} = \{xy^{-1}: x \in A, y \in B\}$ and $AB = \{xy: x \in A, y \in B\}$ contain nonempty open sets.*

**Proof.** We shall prove only that $AB^{-1}$ contains a nonempty open set

Assume without loss of generality that no nonempty open set of $G$ is of first category. For, if some open set $U$ is of first category, by the Banach category theorem (see [2, p. 62]), $G$, which is the union of all translates of $U$, is of first category.

Let $A = G_1 \Delta P_1$ and $B = G_2 \Delta P_2$, where $G_1, G_2$ are nonempty open sets and $P_1, P_2$ are of first category. We shall show that $G_1G_2^{-1}$ which is a nonempty open set, is contained in $AB^{-1}$.

Take any $x \in G_1G_2^{-1}$. Then

$$(*) \qquad xB \cap A \supset (xG_2 \cap G_1) - (xP_2 \cup P_1)$$

and $xG_2 \cap G_1$ is a nonempty open set Since $xP_2 \cup P_1$ is of first category, and since no nonempty open set is of first category, the right-hand side of $(*)$ is nonempty, i.e., $xB \cap A \neq \emptyset$. Thus $x \in AB^{-1}$.

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