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# ON THE EXISTENCE OF A SOLUTION OF $f(x) = kx$ FOR A CONTINUOUS NOT NECESSARILY LINEAR OPERATOR

ANA I. ISTRĂŢESCU AND VASILE I. ISTRĂŢESCU

ABSTRACT. In a recent paper, S. Venkateswaran has asserted that $f(x) = kx$ has a solution when $|k|$ is sufficiently large. In the paper a counterexample to this assertion is given, and it is indicated when the assertion is true.

Let $E$ be a Banach space and $f$ a continuous operator on $E$; i.e. $f$ is a continuous, not necessarily linear mapping from $E$ into $E$. In a recent note, S. Venkateswaran has proved that $f(x) = kx$ has a solution when $|k|$ is sufficiently large.

In what follows we give a counterexample to this assertion.

Let $E$ be the space of all convergent sequences to zero, and define the mapping

$$f(x) = (1, |x_2|^{1/2}, |x_3|^{1/2}, \cdots, |x_n|^{1/2}, \cdots) + (1, 1/2, 1/3, \cdots)$$

which is continuous and nonlinear.

Suppose now that for some $k$ there exists $x_0$ such that $f(x_0) = kx_0$. If $x_0 = (x_1, x_2, \cdots)$, we obtain $kx_1 = 2$ and, thus, $k \neq 0$, and for all $i \geq 2$,

$$kx_i = x_i^{1/2} + 1/i,$$

which gives that $|kx_i| \geq |x_i|^{1/2}$, which implies that $|kx_i^{1/2}| \geq 1$, which represents a contradiction.

For generalization of the Altman fixed point theorem, we refer to [2], [3] and remark that the Theorem of [1] is true when $f$ is completely continuous (compact mapping) or condensing (densifying).

The authors are indebted to Professor L. A. Rubel for calling attention to the fact that the error in Venkateswaran's paper can be traced to the book of Schwartz which misstates Altman's theorem; the assertion and proof are correct if "completely continuous" is substituted for "continuous".

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INSTITUTE OF MATHEMATICS, BUCHAREST, CALEA GRIVITEI 21, ROMANIA