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SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

EVERYWHERE DIFFERENTIABLE FUNCTIONS AND THE DENSITY TOPOLOGY

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Katznelson and Stromberg [3] have recently given a relatively simple proof of the existence of an everywhere differentiable function which is not monotonic in any interval. Our purpose is to show the connection between this property and the density topology.

The density topology for the reals is completely regular [1], [2]. Since countable sets are closed in this topology, for each countable S and $\xi \notin S$ there is an approximately continuous f such that $0 \leq f(x) \leq 1$, $f(\xi) = 1$ and $f(x) = 0$ for each $x \in S$. Let A and B be disjoint countable sets each dense in the reals. They may be enumerated $A = \{a_n\}$, $B = \{b_n\}$. For each n, let f_n be approximately continuous, $0 \leq f_n(x) \leq 1$, $f_n(a_n) = 1$, and $f_n(x) = 0$ for each $x \in B$, and let g_n be approximately continuous, $0 \leq g_n(x) \leq 1$, $g_n(b_n) = 1$, and $g_n(x) = 0$ for each $x \in A$. The function

$$f = \sum_{n=1}^{\infty} 2^{-n} f_n - \sum_{n=1}^{\infty} 2^{-n} g_n$$

is bounded, approximately continuous, positive on A and negative on B. Let F be an indefinite integral of f. Then F is everywhere differentiable and $F' = f$. So F is not monotonic in any interval.

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