ON A PROBLEM OF J. L. TAYLOR
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ABSTRACT. Let $S$ be the structure semigroup of a measure algebra $M(G)$ and $K$ be the union of all maximal groups of $S$. Taylor proposed the following problem: Are there L. C. A. groups $G$ with nontrivial measures concentrated on $K$? The purpose of this paper is to give a positive solution to this problem.

Let $G$ be a locally compact abelian group with dual group $\hat{G}$. We denote by $M(G)$ the Banach algebra of all bounded regular Borel measures on $G$ under convolution multiplication and total variation norm. In [2], Taylor showed that there is a compact topological semigroup $S$, called the structure semigroup of $M(G)$, and an order preserving isometry-isomorphism $\theta$ of $M(G)$ into $M(S)$ such that:

1. $\theta(M(G))$ is a weak*-dense $L$-subalgebra of $M(S)$;
2. the maximal ideal space of $M(G)$ is identified with $\hat{S}$, the set of all continuous semicharacters on $S$, and the Gel'fand transform of $\mu \in M(G)$ is given by $\hat{\mu}(f) = \int S f d\mu$ for $f \in \hat{S}$. $\{\hat{\mu}(f); f \in \hat{S}\}$ is called the spectrum of $\mu$.

$\mu$ is called symmetric if $\hat{\mu^*}(f) = \hat{\mu}(f)$ for every $f \in \hat{S}$. Let $K$ be the union of all maximal groups $\{K_p\}_{p \in P}$ of $S$. Then $K = \{x \in S; |f(x)| = 1 \text{ or } 0 \text{ for every } f \in \hat{S}\}$.

Definition. Let $M_K(G)$ be the set of all $\mu \in M(G)$ such that $\theta\mu$ is concentrated on $K$ but $\theta|\mu|(K_p) = 0$ for every $p \in P$.

In [3], Taylor proposes the following problem concerning $M_K(G)$.

Problem. Are there L.C.A. groups $G$ for which $M_K(G) \neq 0$?

The purpose of this paper is to show the existence of a L.C.A. group $G$ such that $M_K(G) \neq 0$. The following is our main

Theorem. Let $\overline{R}$ be the Bohr compactification of the real line $R$. Then there exists nonzero $\mu \in M_K(\overline{R})$ so that $\mu$ is a positive symmetric measure and the spectrum of $\mu$ is a countable set.

We put $\Lambda_n = \{(\alpha_0, \alpha_1, \ldots, \alpha_n); \alpha_0 = 0, \alpha_i = 0 \text{ or } 1 (i = 1, 2, \ldots, n)\}$ $(n = 0, 1, 2, \ldots)$ and $\Lambda = \bigcup_{n=0}^{\infty} \Lambda_n$. For $\alpha \in \Lambda$, we put $|\alpha| = n$ if $\alpha \in \Lambda_n$.

Lemma 1. There exists a countable family $\{E_\alpha\}_{\alpha \in \Lambda}$ such that $E_\alpha$ is a
subset of \( R (\alpha \in \Lambda) \) satisfying the following conditions:

1. \( E_{\alpha} \subseteq E_{\alpha,0} \) and \( E_{\alpha} \subseteq E_{\alpha,1} \) for \( \alpha \in \Lambda \);
2. \( E_{\alpha,0} \setminus E_{\alpha} \neq \emptyset \) and \( E_{\alpha,1} \setminus E_{\alpha} \neq \emptyset \) for \( \alpha \in \Lambda \);
3. for \( \alpha, \beta \in \Lambda \), \( E_{\alpha} \cap E_{\beta} = E_{\alpha,0,\alpha_1,\ldots,\alpha_j} \) if \( \alpha_1 = \beta_1, \ldots, \alpha_j = \beta_j \) and \( \alpha_{j+1} \neq \beta_{j+1} \);
4. \( \bigcup_{\alpha \in \Lambda} E_{\alpha} \) is an independent set.

Proof. Since \( R \) contains an infinite independent set, it is easy to construct such a family.

We denote by \( H_{\alpha} \) the subgroup of \( R \) generated by \( E_{\alpha} (\alpha \in \Lambda) \). The following lemma is clear by Lemma 1.

Lemma 2. The countable family \( \{ H_{\alpha} \}_{\alpha \in \Lambda} \) has the following properties:

1. \( H_{\alpha} \subseteq H_{\alpha,0} \) and \( H_{\alpha} \subseteq H_{\alpha,1} \) for \( \alpha \in \Lambda \);
2. \( H_{\alpha,0}/H_{\alpha} \) and \( H_{\alpha,1}/H_{\alpha} \) are infinite subgroups for \( \alpha \in \Lambda \);
3. for \( \alpha, \beta \in \Lambda \), \( H_{\alpha} \cap H_{\beta} = H_{\alpha,0,\alpha_1,\ldots,\alpha_j} \) if \( \alpha_1 = \beta_1, \ldots, \alpha_j = \beta_j \) and \( \alpha_{j+1} \neq \beta_{j+1} \).

Let \( G_{\alpha} \) be the annihilator in \( \overline{R} \) of \( H_{\alpha} (\alpha \in \Lambda) \). We put \( G_{\alpha} + G_{\beta} = \{ x + y; x \in G_{\alpha}, y \in G_{\beta} \} \); then \( G_{\alpha} + G_{\beta} \) is a compact subgroup. The following lemma is clear by Lemma 2.

Lemma 3. \( \{ G_{\alpha} \}_{\alpha \in \Lambda} \) is a family of compact subgroups of \( \overline{R} \) and has the following properties:

1. \( G_{\alpha} \supseteq G_{\alpha,0} \) and \( G_{\alpha} \supseteq G_{\alpha,1} \) for \( \alpha \in \Lambda \);
2. \( G_{\alpha,0}/G_{\alpha} \) and \( G_{\alpha,1}/G_{\alpha} \) are compact infinite groups for \( \alpha \in \Lambda \);
3. for \( \alpha, \beta \in \Lambda \), \( G_{\alpha} + G_{\beta} = G_{\alpha,0,\alpha_1,\ldots,\alpha_j} \) if \( \alpha_1 = \beta_1, \ldots, \alpha_j = \beta_j \) and \( \alpha_{j+1} \neq \beta_{j+1} \).

For a compact subgroup \( X \subset \overline{R} \), we denote by \( m_X \) the normalized Haar measure on \( X \). We can consider \( m_X \in \mathcal{M}(\overline{R}) \). We put \( \mu_n = (\frac{1}{2})^n \sum_{\alpha \in \Lambda} m_{G_{\alpha}} \) \((n = 0, 1, \ldots) \). Then \( \mu_n \in \mathcal{M}(\overline{R}), \mu_n \geq 0 \) and \( \| \mu_n \| = 1 \). For \( \mu \in \mathcal{M}(\overline{R}) \), we denote by \( \widehat{\mu} \) the Fourier-Stieltjes transform of \( \mu \). By the definition of \( \{ \mu_n \}_{n=0}^{\infty} \) and Lemma 2, we get

Lemma 4. \( \{ \mu_n \}_{n=0}^{\infty} \) has the following properties:

1. If \( \gamma \in H_0 \), then \( \hat{\mu}_n(\gamma) = 1 \) for \( n = 0, 1, 2, \ldots \);
2. if \( \gamma \in H_{\alpha,0,a_1,\ldots,a_{k-1}} \setminus H_{\alpha,0,a_1,\ldots,a_{k-1}} \), then \( \hat{\mu}_n(\gamma) = (\frac{1}{2})^k \) for \( n \geq k \) and \( \mu_n(\gamma) = 0 \) for \( n < k \);
3. if \( \gamma \in R \setminus H_{\alpha} \) for every \( \alpha \in \Lambda \), then \( \hat{\mu}_n(\gamma) = 0 \), \( n = 0, 1, 2, \ldots \).

By Lemma 4, \( \{ \mu_n \}_{n=0}^{\infty} \) has only one weak*-cluster point \( \mu \) in \( \mathcal{M}(\overline{R}) \) and has the following properties.

Lemma 5. \( \begin{array}{l}
1. \mu \in \mathcal{M}(\overline{R}), \mu \geq 0 \text{ and } \| \mu \| = 1; \\
2. \text{if } \gamma \in H_0 \text{ then } \hat{\mu}(\gamma) = 1. \\
\end{array} \)
(3) if \( \gamma \in \mathcal{H}_{a_0, a_1, \ldots, a_{k-1}, a_k} \), then \( \hat{\mu}(\gamma) = (\gamma')^k \);
(4) if \( \gamma \in \mathbb{R} \setminus H_a \) for every \( a \in \Lambda \), then \( \hat{\mu}(\gamma) = 0 \).

For \( a \in \Lambda \), we put \( \Lambda_n^a = \{ \beta \in \Lambda_n; \alpha_0 = \beta_0, \ldots, a|a| = \beta|a| \} \) for \( n \geq |a| \)
and \( \Lambda_n^a = \bigcup_{n>|a|} \Lambda_n^a \). We put \( \mu_n^a = \sum_{\beta \in \Lambda_n^a} (\gamma')^n m_{\beta} \) for \( n \geq |a| \). Then \( \mu_n^a \geq 0 \), \( \|\mu_n^a\| = (\gamma')^{|a|} \) and \( \mu_{n+|a|}^a = |a| \) has only one weak*-cluster point \( \mu^a \) in \( M(\mathbb{R}) \), and \( \mu_{n+|a|}^a \) has the following properties.

**Lemma 6.**
(1) \( \mu_n^a = \sum_{\beta \in \Lambda_n^a} \mu_{|a|}^a \);
(2) if \( \gamma \in H_a \), then \( \hat{\mu}_n^a(\gamma) = (\gamma')^{|a|} \);
(3) for \( \gamma \in \mathbb{R}^\alpha \), \( \mu_n^a \) has the following properties.

- if \( \beta \in \Lambda_n^a \), \( \hat{\mu}_n^a(\gamma) = (\gamma')^k \) if \( (\beta, \beta_1, \ldots, \beta_k) \in \Lambda_n^a \),
- \( \hat{\mu}_n^a(\gamma) = 0 \) if \( (\beta, \beta_1, \ldots, \beta_k) \notin \Lambda_n^a \),
(4) if \( \gamma \in \mathbb{R} \setminus H_a \) for every \( a \in \Lambda \), then \( \hat{\mu}_n^a(\gamma) = 0 \).

For a compact subgroup \( X \subset \mathbb{R} \), there exists the strongest L.C.A. group topology on \( \mathbb{R} \) such that \( X \) is an open compact subgroup of \( \mathbb{R} \). We denote by \( \mathbb{R}_X \) the resulting L.C.A. topological group. We may consider \( M(\mathbb{R}_X) \subset M(\mathbb{R}) \). For \( \lambda_1, \lambda_2 \in M(\mathbb{R}) \), we denote by \( \lambda_1 \perp \lambda_2 \) if \( \lambda_1 \) is mutually singular with \( \lambda_2 \). For \( \lambda \in M(\mathbb{R}) \) and a subset \( N \subset M(\mathbb{R}) \), we denote by \( \lambda \perp N \) if \( \lambda \perp \nu \) for every \( \nu \in N \).

**Lemma 7.**
(1) \( \mu = \sum_{\alpha \in \Lambda_k} \mu^\alpha \) for every positive integer \( k \);
(2) \( \mu^a \in M(\mathbb{R}_{G_a}) \), and \( \mu^\beta \perp M(\mathbb{R}_{G_a}) \) for \( \beta \neq a, |\beta| = |a| \);
(3) \( \alpha \neq \beta \),
\[
\mu^\alpha \star \mu^\beta = (\gamma')^{|a|} (\gamma')^{|\beta|} m_{G_a} \alpha_{a_0, a_1, \ldots, a_j}
\]
if \( \alpha_1 = \beta_1, \ldots, \alpha_j = \beta_j \) and \( \alpha_{j+1} \neq \beta_{j+1} \).

**Proof.** By (3) of Lemmas 2, 5, and 6, we have \( \hat{\mu} = \sum_{\alpha \in \Lambda_k} \hat{\mu}^\alpha \) for every integer \( k \), that is \( \mu = \sum_{\alpha \in \Lambda_k} \mu^\alpha \). For \( \alpha \neq \beta \) such that \( \alpha_1 = \beta_1, \ldots, \alpha_j = \beta_j \) and \( \alpha_{j+1} \neq \beta_{j+1} \), we have \( \hat{\mu}^\alpha \star \hat{\mu}^\beta(\gamma) = \hat{\mu}^\alpha(\gamma) \hat{\mu}^\beta(\gamma) = (\gamma')^{|a|} (\gamma')^{|\beta|} \) if \( \gamma \in \mathcal{H}_{a_0, \ldots, a_j} \),
\[
= 0
\]
if \( \gamma \notin \mathcal{H}_{a_0, \ldots, a_j} \).

by (3) of Lemmas 2 and 6. This shows that
\[
\mu^\alpha \star \mu^\beta = (\gamma')^{|a|} (\gamma')^{|\beta|} m_{G_a} \alpha_{a_0, a_1, \ldots, a_j}.
\]
Let \( \phi \) be a canonical homomorphism of \( \mathbb{R} \) onto \( \mathbb{R}/G_a \), and for \( \lambda \in M(\mathbb{R}) \) we put \( \hat{\lambda}(E) = \hat{\lambda}(\phi^{-1}(E)) \) for every Borel set \( E \) of \( \mathbb{R}/G_a \). Then \( \hat{\lambda} \in M(\mathbb{R}/G_a) \) and
\[ \lambda(y \circ \phi) = \lambda(y) \quad \text{for} \quad y \in \overline{R/G_a} = H_a. \] If \( y \in H_a \) (\( (\alpha_0, \alpha_1, \ldots, \alpha_k) \)) then \( y \in H_{\alpha_0, \alpha_1, \ldots, \alpha_j} \neq \emptyset \) for some \( 0 \leq j \leq k \) and \( \hat{\lambda}(y) = (1/2)^j \) by (3) of Lemma 5. Then we have

\[ \mu = \frac{1}{2} m_{D_{\alpha_0}} \cdot (\frac{1}{2})^2 m_{D_{a_1}} \cdot \cdots \cdot (\frac{1}{2})^k m_{D_{a_{j-1}}} \cdot (\frac{1}{2})^k \delta_0, \]

where \( D_{\alpha_0, \alpha_1, \ldots, \alpha_j} \) is the annihilator in \( \overline{R/G_a} \) of \( H_{\alpha_0, \alpha_1, \ldots, \alpha_j} \subset H_a \)
\( m_{D_{\alpha_0, \alpha_1, \ldots, \alpha_j}} \) is the normalized Haar measure on \( D_{\alpha_0, \alpha_1, \ldots, \alpha_j} \) and \( \delta_0 \) is the point measure at \( 0 \in \overline{R/G_a} \). Since \( H_a/H_{\alpha_0, \alpha_1, \ldots, \alpha_j} \) is an infinite group, \( m_{D_{\alpha_0, \alpha_1, \ldots, \alpha_j}} \) is a continuous measure on \( \overline{R/G_a} \). Since \( \mu_n (n \geq |\alpha|) \) is concentrated on \( G_a, \mu^\alpha \) is concentrated on \( G_a \) and \( \mu^\alpha = (\frac{1}{2})^k \delta_0 \). Thus
\[ \Sigma_{\beta \neq \alpha} \mu^\beta \mu^\alpha \] is a continuous measure on \( \overline{R/G_a} \) and we have \( \mu^\alpha \in M(\overline{R/G_a}) \) and \( \mu^\beta \perp M(\overline{R/G_a}) \) for \( \beta \neq \alpha \) and \( |\beta| = |\alpha| \).

Remark. By (2) of Lemma 7, \( \mu^\alpha \perp \mu^\beta \) if \( \alpha \neq \beta \) and \( |\alpha| = |\beta| \).

By Lemma 7, we have

**Proposition 1.** \( \theta \mu(K_p) = 0 \) for every maximal group \( K_p \) of \( S \).

**Proof.** Suppose \( \theta \mu(K_p) \neq 0 \) for a maximal group \( K_p \) of \( S \). Then there is a positive integer \( n \) such that \( (\frac{1}{2})^n < \theta \mu(K_p) \). By (1) of Lemma 7, there is \( \alpha \in \Lambda_n \) such that \( \theta \mu^\alpha(K_p) \neq 0 \). By (2) of Lemma 7, we have \( \theta \mu^\beta(K_p) = 0 \) for every \( \beta \in \Lambda_n (\beta \neq \alpha) \). So we have \( \theta \mu(K_p) = \theta \mu^\alpha(K_p) \leq \|\mu^\alpha\| = (\frac{1}{2})^n \), a contradiction.

For \( f \in \hat{S} \) and \( f^2 = f \), we put \( S_0(f) = \{ x \in S ; f(x) = 0 \} \), \( S_1(f) = \{ x \in S ; f(x) = 1 \} \), and \( M(S_1(f)) = \{ \mu \in M(R); \theta \mu \text{ is concentrated on } S_j(f) \} \) (\( j = 0, 1 \)). Then \( M(S_1(f)) \) is an L-ideal of \( M(\overline{R}) \) and \( M(S_1(f)) \) is an L-subalgebra [2].

**Lemma 8.** Let \( f \in \hat{S} \) such that \( f^2 = f \) and \( \hat{\mu}(f) \neq 0 \). Then there exists \( \alpha \in \Lambda \) such that:

1. \( \mu^\alpha \in M(S_1(f)) \);
2. \( \mu^\beta \in M(S_0(f)) \) for \( \beta \neq \alpha \) and \( |\beta| = |\alpha| \).

**Proof.** Since \( \hat{\mu}(f) \neq 0 \), we can decompose \( \mu = \lambda_1 + \lambda_2 \) (\( \lambda_2 \neq 0 \)), where \( \lambda_1 \in M(S_1(f)) \) and \( \lambda_2 \in M(S_0(f)) \). Suppose that \( \mu_n \in M(S_0(f)) \) for every integer \( n \). For some integer \( n_0 \) such that \( (\frac{1}{2})^{n_0} < \|\lambda_2\| \), there exists \( \chi, \nu \in \Lambda_{n_0} \) such that \( \mu^\chi \neq \lambda_2 \) and \( \mu^\nu \neq \lambda_2 \). Because, \( \mu^\chi \mu^\nu \) for \( \chi, \nu \in \Lambda_{n_0} \) and \( \chi \neq \nu \), by the remark of Lemma 7, and \( \|\mu^\chi\| = (\frac{1}{2})^{n_0} \) for every \( \chi \in \Lambda_{n_0} \).

By Lemma 7, we have

\[ \mu^\chi \mu^\nu = (\frac{1}{2})^{n_0} \|\chi\| \|\nu\| \mu^\chi \mu^\nu, \]

where \( \chi_1 = \nu, \ldots, \chi_j = \nu \) and \( \chi_{j+1} \neq \nu \), and \( \mu^\chi \mu^\nu \in M(S_0(f)) \). Since \( \mu^\chi \neq \lambda_2 \) and \( \mu^\nu \neq \lambda_2 \), we have \( \mu^\chi \mu^\nu \neq \lambda_2 \ast \lambda_2 \). Since \( \lambda_2 \ast \lambda_2 \in M(S_1(f)) \), we have \( \mu^\chi \mu^\nu \perp M(S_0(f)) \), a contradiction. Thus there exists an integer \( n \) such that \( \mu_n \notin M(S_0(f)) \). Let \( n_1 \) be the smallest integer such that \( \mu_n \notin M(S_0(f)) \). Then there exists \( \alpha \in \Lambda_{n_1} \) such that \( \mu^\alpha \in M(S_1(f)) \) and \( m_{G_{\beta}} \in \)
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\[ M(S_0(f)) \] for \( \beta \in \Lambda_{n_1} \) and \( \beta \neq \alpha \), by (3) of Lemma 3. Since \( M(\overline{R}_{G_{\alpha}}) \subseteq M(S_1(f)) \), we have \( \mu^\alpha \in M(S_1(f)) \) by (2) of Lemma 7. Suppose that \( \mu^\beta \notin M(S_0(f)) \) for some \( \beta \in \Lambda_{n_1} \) and \( \beta \neq \alpha \). Then we have \( \mu^\beta \ast \mu^\alpha \notin M(S_0(f)) \).

By (3) of Lemma 7, we have

\[ \mu^\beta \ast \mu^\alpha = (\frac{1}{2^2}) |\beta| + \frac{1}{2} m_{\alpha_{0,1,\ldots,n_1}} \]

where \( \alpha_1 = \beta_1, \ldots, \alpha_{i_j} = \beta_{j_i} \) and \( \alpha_{j+1} \neq \beta_{j+1} \), and \( j < |\alpha| = n_1 \). This shows that \( \mu^\beta \ast \mu^\alpha \notin M(S_0(f)) \), a contradiction. Thus we complete this lemma.

For \( f \in \mathcal{S} \), \( f \geq 0 \), there exists \( g_\xi \in \mathcal{S} \), \( g_\xi = g_\xi \) such that \( V_S(\mathcal{S}_j(g_\xi)) = M(\mathcal{S}_j(f/\mathcal{S}_j(g_\xi))) \), where \( \mathcal{S}_j(f) = \{ x \in \mathcal{S}; f(x) = 1 \} \) and \( M(\mathcal{S}_j(f)) = \{ \mu \in M(\overline{R}); \theta \mu \text{ is concentrated on } \mathcal{S}(f) \} \) [2].

**Proposition 2.** \( \theta \mu \) is concentrated on \( K \).

**Proof.** Let \( f \in \mathcal{S} \) such that \( f \geq 0 \), \( f \neq f \) and \( \mu(f) \neq 0 \). Let \( f = h_\xi \cdot f \) be the polar decomposition of \( f \), where \( h_\xi = h_\xi \in \mathcal{S} \) [2, Lemma 3.3]. Then \( \nu(h_\xi) \neq 0 \). By Lemma 8, there exists \( \alpha \in \Lambda \) such that \( \mu^\alpha \in M(\mathcal{S}_1(\mathcal{S}_j(g_\xi))) \) and \( \mu^\beta \in M(\mathcal{S}_0(\mathcal{S}_j(g_\xi))) \) for \( \beta \neq \alpha \) and \( |\beta| = |\alpha| \). Since \( M(\overline{R}_{G_{\alpha}}) \subseteq M(\mathcal{S}_1(\mathcal{S}_j(g_\xi))) \) and \( m_{\alpha_{0,1,\ldots,n_1}} \in M(\mathcal{S}_1(\mathcal{S}_j(g_\xi))) \), we have \( M(\overline{R}_{G_{\alpha}}) \subseteq M(\mathcal{S}_1(\mathcal{S}_j(g_\xi))) \) [3]. Thus we complete the proof of this proposition.

**Proposition 3.** \( \mu \) is a symmetric measure and \( \mu \) has a countable spectrum.

**Proof.** Since \( \mu \geq 0 \), we have \( \mu^* = \mu \). Let \( f \in \mathcal{S} \). By the proof of Proposition 2, there exists \( \alpha \in \Lambda \) such that \( \mu(\mathcal{S}_j(f)) = \mu^\alpha(\mathcal{S}_j(f)) \). Since \( \mu^\alpha \in M(\overline{R}_{G_{\alpha}}) \) and \( \mathcal{S}(\mathcal{S}_j(f)) = \|\lambda\| \) for every positive \( \lambda \in M(\overline{R}_{G_{\alpha}}) \), there exists \( \gamma \in \overline{R}_{G_{\alpha}} \) such that \( \tilde{\mu}(\mathcal{S}_j(f)) = \tilde{\mu}(\gamma) \) [2]. Since \( \mu^\alpha \in M(G_{\alpha}) \), there exists \( \eta \in \hat{G}_{\alpha} \subseteq R \) such that \( \tilde{\mu}(\mathcal{S}_j(f)) = \tilde{\mu}(\gamma) = \tilde{\mu}(\eta) \). By Lemma 6, we have

\[ \{ \mu^\alpha(\eta); \eta \in R; \xi \in \mathcal{S}_1(\mathcal{S}_j(g_\xi)) = 0, (\frac{1}{2})^{|\alpha|}, (\frac{1}{2})^{|\alpha|+1}, (\frac{1}{2})^{|\alpha|+2}, \ldots \}. \]

Thus we have \( \{ \mu(\mathcal{S}_j(f)); f \in \mathcal{S} \} = \{ 0, 1, \frac{1}{2}, (\frac{1}{2})^2, (\frac{1}{2})^3, \ldots \} \). This shows that \( \mu \) is symmetric and has a countable spectrum.

By Propositions 1, 2 and 3, we have our Theorem.

**Corollary.** There is a compact metrizable abelian group \( G \) and a nonzero symmetric measure \( \mu \in M_K(G) \) so that the spectrum of \( \mu \) is a countable set.

**Proof.** We may assume that \( E_{\alpha} \) is a countable set \( \alpha \in \Lambda \) in Lemma 1. Let \( H \) be the subgroup generated by \( \{ H_{\alpha}; \alpha \in \Lambda \} \); then \( H \) is a countable subgroup of \( R \). Let \( H^\perp \) be the annihilator of \( H \) in \( \overline{R} \); then \( H^\perp \) is a compact subgroup of \( \overline{R} \). Since \( H = \overline{R}/H^\perp \), \( \overline{R}/H^\perp \) is a compact metrizable group. Then we can construct \( \mu \in M(\overline{R}/H^\perp) \), which has the properties of this corollary, in the same way as in the proof of our Theorem.
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