ERRATUM TO "GENERALIZED RELATIVE DIFFERENCE SETS"

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ABSTRACT. This note gives the correction of a theorem previously published. A counterexample is also given for the theorem as originally stated.

In [1], a nonexistence theorem was given for a normal $(0, 1)$-matrix $A$ with three distinct characteristic roots such that $A^TA = c_0I - c_1(I_m \otimes J_n)$. However, part (i) of Theorem 1.5 as stated is incorrect. A counterexample is the cyclic relative difference set $R(13, 2, 9, 3)$ whose elements are $(2, 4, 6, 7, 10, 11, 12, 18, 21)$. For this set, $m$ is odd and the Hilbert symbol $(3, 26)_3 = -1$.

In the notation of [1], let $W_1$ be the space of characteristic (column) vectors of $B = A^TA$ associated with $\theta_1 = c_0 - nc_1$. For $1 < j < m$, let $\alpha_j$ be the vector of size $mn$ with $+1$ in positions $1, 2, \ldots, n$, with $-1$ in positions $jn + 1, \ldots, jn + n$, and with zeros elsewhere. Then $\{\alpha_j | j = 1, \ldots, m - 1\}$ is a basis for $W_1$. Let $G = ((\alpha_i, \alpha_j))_{1 \leq i, j \leq m-1}$. Then $G = n(I_{m-1} + J_{m-1})$. Thus $q_1 = det G = n^{m-1}m$, instead of $q_1 = mn$ as stated in [1]. Hence Theorem 1.5(i) should read: If $m$ is odd, then $(c_0 - nc_1, (-1)^{(m-1)/2})_p = +1$ for all primes $p$.

The Bruck-Ryser Theorem applied to $A^*$ (see [1, Theorem 1.6]) says in this case that $(c_0 - nc_1, (-1)^{(m-1)/2}nc_1)_p = +1$ for all primes $p$. These two conditions are equivalent if and only if $(c_0 - nc_1, mnc_1)_p = +1$ for all primes $p$. But $c_0 - nc_1 = c_0^2 - mnc_1$. Put $x = 1/c_0 = y$. Then $x^2(c_0^2 - mnc_1) + y^2(mnc_1) = 1$, implying that $(c_0 - nc_1, mnc_1)_p = +1$ for all primes $p$. Hence the Bruck-Ryser Theorem applied to $A^*$ is indeed equivalent to the correct form of Theorem 1.5(i).

REFERENCES


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