GROSS' ABSTRACT WIENER MEASURE
ON C[0, ∞)

H. C. FINLAYSON

Abstract. Classical Wiener measure on C[0, ∞) is obtained by the construction of Gross' abstract Wiener measure on a suitable Banach subspace of C[0, ∞).

In two other papers [2], [3] three classical Wiener measures were shown to be special cases of Gross' abstract measure. In each of those cases the space on which the measure was given was a Banach space with an obvious supremum norm. Since C[0, ∞) is not a Banach space with the supremum norm, the problem of constructing the classical measure on it by Gross' method needs a slight modification.

Let C' be the subspace of absolutely continuous functions on [0, ∞) with square integrable derivative. C' with the inner product

\[(x, y) = \int_0^\infty x'(t)y'(t)\,dt\]

is the Hilbert space to be used in the construction of abstract Wiener measure. It is easy to show for \(x \in C'\) that \(\|x\|\) exists, where

\[\|x\| = \sup_{t \in [0,\infty)} \sqrt{(2/\Pi)} \left| \int_0^t \left[ x'(s)/\sqrt{(1 + s^2)} \right] ds \right|,\]

and is a norm on C'. Also, by use of the C. O. N. set of functions

\[\left\{ F_n(t) = \sqrt{(2/\Pi)} \int_0^t [1/\sqrt{(1 + s^2)}] h_n(2 \arctan s/\Pi) \, ds \right\}\]

where \(h_n(s)\) is the \(n\)th Haar function on [0, 1], [1, p. 16], it is easy to parallel the argument in [2] to show that \(\|\|\) is a measurable norm. The completion of C' in this norm is the subspace B of C[0, ∞) for which \(\int_0^\infty [1/\sqrt{(1 + s^2)}] dx(s)\) converges, and it is on B that the abstract measure is given.

Now from the law of the iterated logarithm in classical Wiener space [4] (which implies \(x(t) = O(\sqrt{t \ln t})\) a.e.), and from the definition of B above there follows that B has measure one in classical Wiener space. Finally, a consideration of linear functionals on B which are integrals of step functions shows that the abstract measure assigned to \(\{x \in B: x(t_i) \in [a_i, b_i]: i = 1, 2, \ldots, n\}\) is the same as the classical measure assigned to the same set. Thus

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the classical measure is obtained from the abstract measure by the enlargement of $B$ to $C[0, \infty)$ and the assignment of measure zero to subsets of $C[0, \infty) \setminus B$.

**REFERENCES**


DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MANITOBA, WINNIPEG, MANITOBA, CANADA