ON A CHARACTERIZATION OF BARRELLED SPACES

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Abstract. We prove that a l.c. space $E$ is barrelled if every closed graph linear map: $E \to C(X)$ is continuous; $X$ compact $T_2$. This generalizes Mahowald's criterion, but the main contribution is the simplicity of the proof.

We give a simple proof of (a strengthened version of) Mahowald's characterization of barrelled spaces which seems to have been overlooked in the standard texts.

Let $E$ be a locally convex space such that for every compact Hausdorff space $H$ and linear map $T: E \to C(H)$ with closed graph, $T$ must be continuous. Let $B$ be a barrel in $E$, $B^0$ its (absolute) polar in $E'$. Let $P$ be $B^0$ with the weak* topology $\sigma(E', E)$, $F = C^*(P)$, the Banach space of bounded continuous maps from $P$ to the scalars. Define $T: E \to F$ by $(Tx)(h) = h(x)$ for $x \in E$, $h \in P$. It is routine to verify that $T$ is linear and has closed graph. By hypothesis, $T$ is continuous. With $D$ the unit disc in $F$, $T^{-1}[D] = B^0 \circ = B$ and so $B$ is a neighborhood of $0$. This proves that $E$ is barrelled.

Remark. It should be noted that $P$ is not known to be compact and that the hypothesis is being applied with $H = \beta P$. The referee observes that the above result is equivalent to Mahowald's theorem since every $F \subset C(H)$ for some $H$.

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