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SHORTER NOTES

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A NEW PROOF OF GIESY'S THEOREM ON NON-B-CONVEXITY

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ABSTRACT. A new proof of Giesy's theorem on non-B-convexity is given.

The following theorem was proved by D. P. Giesy in [2, Lemma 6, §1] and has been often used since (see, for example [1]):

A necessary and sufficient condition for non-D-convexity of a normed linear space X is that given $k \geqslant 2$, $\varepsilon > 0$, there exist $x_1, x_2, \ldots, x_k \in S$ such that for each choice of scalars $\alpha_1, \alpha_2, \ldots, \alpha_k$,

$$(1-\varepsilon)\sum_{i=1}^{k} |\alpha_i| \leqslant \left\| \sum_{i=1}^{k} \alpha_i x_i \right\|.$$

It is known that D-convexity is equivalent to B-convexity [2]. In this note a simple proof for the necessity part is given. The sufficiency is trivial, of course, The notation is the same as that of Giesy [2].

PROOF OF THE NECESSITY. Let $k \geqslant 2$ and $0 < \varepsilon < 1$. Let $\delta = k^{-1}\varepsilon$.

$$X \text{ is not } D\text{-convex} \Rightarrow X \text{ is not } D, x, \delta\text{-convex}$$

$$\Rightarrow \exists x_1, x_2, \ldots, x_k \in S \ni \text{ for any } \lambda_1, \lambda_2, \ldots, \lambda_k \in D,$$

$$k^{-1}\left\| \sum_{i=1}^{k} \lambda_i x_i \right\| > 1 - \delta.$$

Let $\alpha_1, \alpha_2, \ldots, \alpha_k$ be scalars such that $\sum_{i=1}^{k} |\alpha_i| = 1$. Consider

$$\begin{aligned}\left\| \sum_{i=1}^{k} \alpha_i x_i \right\| &\geqslant \left\| \sum_{i=1}^{k} sg(\alpha_i) x_i \right\| - \left\| \sum_{i=1}^{k} (sg(\alpha_i) - \alpha_i) x_i \right\| \\ &> k(1-\delta) - \sum_{i=1}^{k} |sg(\alpha_i) - \alpha_i| = k - \varepsilon - \sum_{i=1}^{k} (1 - |\alpha_i|) \\ &= k - \varepsilon - (k-1) = 1 - \varepsilon.\end{aligned}$$

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Hence $(1-\varepsilon)\sum_{i=1}^{k}|\alpha_i| \leqslant \|\sum_{i=1}^{k}\alpha_i x_i\|$ for any choice of scalars $\alpha_1, \alpha_2, \ldots, \alpha_k$.

References

1. D. R. Brown, *B-convexity and reflexivity in Banach spaces*, Trans. Amer. Math. Soc. **187** (1974), 69–76.

2. D. P. Giesy, *On a convexity condition in normed linear spaces*, Trans. Amer. Math. Soc. **125** (1966), 114–146; corrections, ibid. **140** (1969), 511–512. MR **34** #4866; **39** #1945.

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