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NOTE ON MAXIMALLY ALMOST PERIODIC GROUPS

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ABSTRACT. A simple proof is given that every locally compact maximally almost periodic group is unimodular.

A topological group G is Maximally Almost Periodic (MAP), if G admits a continuous monomorphism into a compact Hausdorff group, for references, see e.g. [5].

All topological groups here are assumed to be Hausdorff. The purpose of this note is to give a simple proof of the following

THEOREM (H. LEPTIN, L. ROBERTSON [4]). *Every locally compact MAP group is unimodular.*

PROOF. The proof is based on [2, Theorem XV, p. 66]. Using the same notations as in Theorem XV, let G be a locally compact MAP group and U its maximal unimodular subgroup (which always exists and is normal). Suppose that G is the semidirect and not direct product of U and a one-dimensional vector subgroup N. Let G_0 be the component of the identity in G. Then G_0 is connected locally compact MAP and hence, by a structure theorem of Freudenthal and Weil, see e.g. [2], G_0 is a direct product of a compact group C and a vector group N_1. By the maximality of U, it follows that both C and N_1 are subgroups of U and hence so is G_0. However, since G_0 contains N, N is contained in U. This is a contradiction. Suppose that U is open and G/U is a discrete abelian group algebraically isomorphic to a subgroup of reals (hence it is torsion free and contains no nontrivial compact subgroups). Let φ be a continuous monomorphism of G into a compact group $\mathfrak{M}$. Then φ induces an action π of G on $\mathfrak{M}$ such that the transformation group $(\mathfrak{M}, G, \pi)$ is equicontinuous and strongly effective, that is, the transition group $\{\pi^t | t \in G\}$ is a family of equicontinuous mappings of $\mathfrak{M}$ into $\mathfrak{M}$, and if $t \in G$ such that there exists $x \in \mathfrak{M}$ with $xt = (x, t)\pi = x$, then $t = e$, where e is the identity of G, see e.g. [1]. By [1, Theorem 4.38, p. 37], the transformation

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group $(\mathfrak{M}, G, \pi)$ is almost periodic. Now let $x \in \mathfrak{M}$. Then since U is open, there exists a left syndetic subset A of G such that $xA \subseteq xU$ and hence $A \subseteq U$. This shows that U is a syndetic (normal) subgroup of G. Consequently, by [1, Remark 2.03, p. 12], G/U is compact. This is a contradiction. Therefore, G is unimodular.

The proof is completed.

REMARK. In [3], it is proved that a topological group G is MAP if and only if G admits an action π on a compact Hausdorff space X such that the transformation group (X, G, π) is equicontinuous and effective, and this characterization of MAP groups is shown to be quite useful in the study of the structures of MAP groups.

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