ININVARIANCE OF DOMAIN IN BANACH SPACES

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Abstract. A continuous one-one map of the open unit disc of $l^\infty$ onto itself which is not a homeomorphism is constructed.

The Brouwer theorem on invariance of domain states that if $G$ is an open subset of Euclidean space $E$ and $f: G \to E$ is a continuous one-one map, then $f(G)$ is open and $f$ is a homeomorphism. This result has been extended to Banach spaces by Schauder [2] in the case when $f$ is of the form $I + \phi$, $\phi$ being completely continuous, and by Tromba [3] in the case when $f$ is a Fredholm map of index zero.

In general, invariance of domain fails in a Banach space since many Banach spaces are linearly homeomorphic to proper subspaces of themselves; furthermore Klee [1] has constructed a homeomorphism of separable Hilbert space onto a closed half-space. However, if the theorem is weakened so that $f(G)$ is already assumed to be open, does the result then hold in a Banach space? The answer is no as can be seen from the following extremely simple example.

Let $g_n: R \to R$ be the map with the graph

![Graph of function $g_n(x)$]

and let $G: l^\infty \to l^\infty$ be given by

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The derivative of $G$ will then have the form

$$G'(x_1, x_2, \ldots, \varepsilon_1, \varepsilon_2, \ldots) = \left( g'_1(x_1)\varepsilon_1, g'_2(x_2)\varepsilon_2, \ldots \right)$$

and will be a continuous transformation as long as $(g'_1(x_1), g'_2(x_2), \ldots) \in l^\infty$.

REFERENCES

