POSITION OF COMPACT HYPERSURFACES
OF THE n-SPHERE

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Abstract. Let $S^n$ be the Euclidean sphere of dimension $n$. Let $p$ and $q$ be antipodal points on $S^n$, and, for nonnegative $h$, let $C(p, h)$, $C(q, h)$ be the hyperspheres of constant mean curvature $h$ centered at $p$ and $q$, respectively. Then any closed hypersurface in $S^n$ with mean curvature bounded by $h$ must have a point in the 'tropical' region bounded by $C(p, h)$ and $C(q, h)$.

1. Introduction. Let $S^n$ be the $n$-sphere with the standard Riemannian metric induced by inclusion in $\mathbb{R}^{n+1}$. For $p \in S^n$, let $C(p, k)$ be the $(n - 1)$-sphere of constant mean curvature $k$, centered at $p$. Let $D(p, k)$ be the component of $S^n - C(p, k)$ containing $p$. We prove:

1.1 Theorem. Let $M$ be a hypersurface in $S^n$ which is smooth, compact, and without boundary. Let $H$ be the mean curvature function on $M$. If $|H| < k$, then for any two antipodal points $p$ and $q$ in $S^n$, there is a point of $M$ lying in the set $A(p, q, k) = S^n - (D(p, k) \cup D(q, k))$.

Note that the boundary of $A(p, q, k)$ is just $C(p, k) \cup C(q, k)$. If $M$ is minimal, then $H \equiv 0$, and we have

1.2 Corollary. Let $M$ be a compact, oriented, minimal hypersurface without boundary in $S^n$. Then $M$ must intersect each great $(n - 1)$-sphere.

The Corollary may also be proved using methods developed by H. B. Lawson [2].

2. Proof of the Theorem. We prove first the following

Lemma. If $|H| < k$, then there is a point of $M$ lying in $S^n - D(p, k)$.

Proof. Suppose $M$ lies entirely in $D(p, k)$. Since $M$ is compact, then it must also lie in the closure of $D(p, r)$ with $r > k$. Shrink $D(p, r)$ until $C(p, r)$ first touches $M$. Then at some point $m$, $M$ is tangent to $C(p, r)$. Since $M$ is tangent from within $D(p, r)$, it follows that $H(m) \geq r > k$. But this contradicts $|H| < k$.

The Theorem now follows by application of the Lemma to the antipodal points $p$ and $q$.

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