SHORTER NOTES

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SPHERICALLY SYMMETRIC ENTIRE SOLUTIONS OF
\[ \Delta^p u = f(u) \]

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Abstract. A sufficient condition is given for existence of spherically symmetric entire solutions of the equation \( \Delta^p u = f(u) \), \( p \geq 2 \).

Some results on spherically symmetric entire solutions of \( \Delta u = f(u) \) are given in [1], [2]. In particular, it is known that \( \Delta u = e^u \) has no entire solutions for all \( n > 2 \). However, W. Walter [3] obtained the startling result that when \( n > 3, p > 2 \), there do exist spherically symmetric entire solutions of \( \Delta^p u = e^u \). The purpose of this note is to prove the following theorem. For simplicity, we shall state and prove the theorem only for the equation \( \Delta^2 u = f(u) \). The theorem holds true for the equation \( \Delta^p u = f(u) \), \( p \geq 2 \).

Theorem. Suppose that \( f(t) \) is a positive continuously differentiable function with \( f'(t) \geq 0 \) for all \( t \). Suppose \( f \) satisfies the condition

\[ \int_0^\infty \left( \int_0^t f(s) \, ds \right)^{-1/2} \, dt = \infty. \]

Then the equation \( \Delta^2 u = f(u) \) has a spherically symmetric entire solution, where \( \Delta \) is the \( n \)-dimensional Laplacian.

Proof. Let \( r^2 = x_1^2 + x_2^2 + \cdots + x_n^2 \). Then

\[ \Delta u(r) = r^{1-n} \frac{d}{dr} \left( r^{n-1} \frac{du}{dr} \right). \]

Employing standard iteration procedure, one can show that there exists a solution \( u(r) \) satisfying \( \Delta^2 u = f(u) \) in a maximal interval \([0, R]\), with \( u(0) = u'(0) = \Delta u(0) = (\Delta u)'(0) = 0 \). It is easily seen that \( u(r) \to \infty \) as \( r \to R \). Since \( (\Delta u)' > 0 \), from \( \Delta^2 u = f(u) \) it follows that \( (\Delta u)'' < f(u) \). Integrating this inequality twice, we obtain \( \Delta u < 2^{-1} r^2 f(u) \). Since \( u' > 0 \), we have \( u' u'' < 2^{-1} r^2 u' f(u) \). Integrating twice, we obtain

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\int_{-\infty}^{\infty} \left( \int_{0}^{u} f(s) \, ds \right)^{-1/2} \, du < R^2.
\]

\( R \) cannot be finite because of the condition (A).

**References**


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