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HOMOGENEITY BY ISOTOPY

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ABSTRACT. Answered is a question asked by Daverman, Bull. Amer. Math. Soc. **84** (1978), 377–405. It is whether a 2-sphere Σ is tame if each isotopy on Σ extends to an isotopy of E^3?

In [**2**], Daverman asked several interesting questions concerning homogeneity. In particular, he asked if a 2-sphere Σ is tame if each isotopy on Σ extends to an isotopy of E^3? The answer is given by the following theorem.[1]

THEOREM. *If Σ is a 2-sphere which is homogeneous by isotopy, then Σ is tame.*

PROOF. Parameterize some cell on Σ by $I^2 = [0, 1] \times [0, 1]$. Let A be an arc which pierces Σ at $(0, 0)$ and is parameterized by a homeomorphism φ: $[-1, 1] \to A$ with $\varphi(0) = (0, 0) \in I^2$. Let H: $E^3 \times I \to E^3$ be the extension of an isotopy on Σ which moves $\varphi(0)$ to $(1, 0)$ by sliding the left side of the unit square over to the right side in such a manner that, at time t, $(0, t_1)$ is at (t, t_1). Let G be such an extension which moves the bottom of I^2 to the top. Having done this we see that $G(H(\varphi(0), I), I)$ is the unit square I^2 on Σ and we need only to define

$$g\colon G\big(H(\phi(0), I), I\big) \times [-1, 1] \to G(H(A, I), I)$$

by

$$g\big(G\big(H(\varphi(0), t_1), t_2\big), t\big) = G\big(H(\phi(t), t_1)t_2\big).$$

Now by Daverman's "singular regular neighborhoods" [**1**] we are done.

Note that by [**1**], this argument also holds for any $(n-1)$-sphere in E^n, $n \neq 4$.

ADDENDUM. This is the theorem to which Daverman was referring in [**2**, p. 387].

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[1]This theorem is taken from the author's dissertation at the University of Georgia which was written under the direction of Professor James Cantrell.

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