HOMOGENEITY BY ISOTOPY

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Abstract. Answered is a question asked by Daverman, Bull. Amer. Math. Soc. 84 (1978), 377-405. It is whether a 2-sphere $\Sigma$ is tame if each isotopy on $\Sigma$ extends to an isotopy of $E^3$?

In [2], Daverman asked several interesting questions concerning homogeneity. In particular, he asked if a 2-sphere $\Sigma$ is tame if each isotopy on $\Sigma$ extends to an isotopy of $E^3$? The answer is given by the following theorem.

Theorem. If $\Sigma$ is a 2-sphere which is homogeneous by isotopy, then $\Sigma$ is tame.

Proof. Parameterize some cell on $\Sigma$ by $I^2 = [0, 1] \times [0, 1]$. Let $A$ be an arc which pierces $\Sigma$ at $(0, 0)$ and is parameterized by a homeomorphism $\varphi: [-1, 1] \rightarrow A$ with $\varphi(0) = (0, 0) \in I^2$. Let $H: E^3 \times I \rightarrow E^3$ be the extension of an isotopy on $\Sigma$ which moves $\varphi(0)$ to $(1, 0)$ by sliding the left side of the unit square over to the right side in such a manner that, at time $t$, $(0, t)$ is at $(t, t_i)$. Let $G$ be such an extension which moves the bottom of $I^2$ to the top. Having done this we see that $G(H(\varphi(0), I), I)$ is the unit square $I^2$ on $\Sigma$ and we need only to define

$$g: G(H(\varphi(0), I), I) \times [-1, 1] \rightarrow G(H(A, I), I)$$

by

$$g(G(H(\varphi(0), t_1), t_2), t) = G(H(\varphi(t), t_1), t_2).$$

Now by Daverman's "singular regular neighborhoods" [1] we are done.

Note that by [1], this argument also holds for any $(n - 1)$-sphere in $E^n$, $n \neq 4$.

Addendum. This is the theorem to which Daverman was referring in [2, p. 387].
BIBLIOGRAPHY


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