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# RC-CONVERGENCE

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ABSTRACT. The rc-convergence structure is introduced and used to characterize $S$-closed spaces in terms of regular-closed (rc) or regular-open sets. $S$-closed spaces are compared with nearly-compact, quasi-$H$-closed spaces and compact semiregularizations. Weakly-$T_2$ extremally disconnected spaces are embedded into the Fomin $S$-closed extension. For any discrete space, $\beta(X)$ is shown to be $S$-closed and the category of nearly-compact Hausdorff spaces and $\theta$-continuous mappings has the $S$-closed spaces as its projective objects. An explicit example of a noncompact Hausdorff $S$-closed space is constructed. Finally, various mappings which preserve $S$-closedness are investigated.

**1. Introduction.** In [**16**], Travis Thompson utilizing semiopen sets introduced the concept of the $S$-closed space. The major goal of this paper is to show that $S$-closed spaces can be characterized in terms of the more familiar regular-closed (rc) or regular-open subsets of a space $(X, T)$ and to introduce the rc-convergence structure. This pretopological convergence structure is used to improve upon the results in [**16**] and to compare $S$-closed, nearly-compact [**13**], quasi-$H$-closed [**6**] and compact semiregularizations. A few of the results in this paper have been independently obtained by D. Cameron in his investigation of maximal topologies [**2**]. For a nonstandard approach to $S$-closedness, we refer the reader to [**6**].

The following are examples of the major propositions obtained. For $(X, T)$ the following are equivalent: (i) $X$ is $S$-closed; (ii) any cover $\mathcal{C}$ of $X$ by regular-closed sets has a finite subcover; (iii) any family $\mathcal{C}$ of regular-open sets such that $\bigcap \mathcal{C} = \varnothing$ contains a finite $\mathfrak{B} \subset \mathcal{C}$ such that $\bigcap \mathfrak{B} = \varnothing$; (iv) every filter base on $X$ has an rc-accumulation point in $X$; (v) every maximal filter base on $X$ rc-converges. If $X$ is nearly-compact (or quasi-$H$-closed) and extremally disconnected then $X$ is $S$-closed. If $X$ is almost-regular [**12**] and $S$-closed, then $X$ is nearly-compact and extremally disconnected. If $X$ is weakly-$T_2$ and $S$-closed, then $X$ is $H$-closed and extremally disconnected. If $X$ is almost-regular, then $X$ is $S$-closed iff $X$ is nearly-compact and extremally disconnected iff $X_s$ is regular, compact and extremally disconnected. If $X$ is weakly-$T_2$, then $X$ is $S$-closed iff $X$ is quasi-$H$-closed and extremally discon-

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nected iff $X_s$ is compact, Hausdorff and extremally disconnected. In the category of nearly-compact Hausdorff spaces and $\theta$-continuous maps, the $S$-closed spaces are precisely the projective objects. If $X$ is weakly-$T_2$ and extremally disconnected, then $X$ is a dense subspace of the $S$-closed Fomin extension $\sigma(X)$. In the class of all weakly-$T_2$ $S$-closed extensions of a weakly-$T_2$ extremally disconnected space $X$, the Fomin extension $\sigma(X)$ is the almost-projective maximum. Certain $S$-closed preserving maps are discussed and an example of a noncompact Hausdorff $S$-closed space is constructed.

**2. Preliminaries.** Throughout this paper $(X, T)$ or simply $X$ and $(Y, \tau)$ or simply $Y$ denote topological spaces. Moreover, unless otherwise indicated, no extremally disconnected space is assumed Hausdorff. We recall the following definitions, notational conventions and characterizations. For $(X, T)$, the notation $(X, T_s)$ or simply $X_s$ denotes the space generated by the regular-open subsets of $X$, the *semiregularization*. A space $X$ is *nearly-compact* [13] iff $X_s$ is compact iff every maximal filter base $\delta$-converges iff every filter base $\delta$-accumulates. A space is *quasi-H-closed* iff each open cover $\mathcal{C}$ of $X$ has a finite proximate subcover $\mathcal{B}$ (i.e., $X = \mathrm{cl}_X(\bigcup \mathcal{B})$) iff each filter base $\theta$-accumulates iff each maximal filter base is $\theta$-convergent [6]. A space is *almost-regular* [resp., *weakly-$T_2$*] iff $X_s$ is regular [resp., $T_1$]. A set $A \subset X$ is *semiopen* if there exists $G \in T$ such that $G \subset A \subset \mathrm{cl}_X G$. A space $X$ is *S-closed* iff every cover $\mathcal{C}$ by semiopen sets has a finite proximate subcover [16]. For each $x \in X$, $\mathfrak{N}(x)$ is the neighborhood filter. If nonempty $\mathfrak{F} \subset \mathfrak{P}(X)$, then $[\mathfrak{F}]$ denotes the filter base and $\langle \mathfrak{F} \rangle$ the filter on $X$ generated by $\mathfrak{F}$. If $x \in X$, then the filter base $\mathcal{C}(x) = \{\mathrm{cl}_X G | x \in G \in T\}$ determines the pretopological $\theta$-convergence structure as follows: A filter base (or filter) $\mathfrak{F}$ $\theta$-converges to $x \in X$ iff $\langle \mathcal{C}(x) \rangle \subset \langle \mathfrak{F} \rangle$. The neighborhood filter $\mathfrak{N}_s(x)$ for $X_s$ determines the $\delta$-convergence structure as follows: A filter base (or filter) $\mathfrak{F}$ $\delta$-converges to $x \in X$ iff $\mathfrak{N}_s(x) \subset \langle \mathfrak{F} \rangle$. For any two filter bases $\mathfrak{F}$, $\mathfrak{G}$ on $X$, $\mathfrak{F} \perp \mathfrak{G}$ [resp., $\mathfrak{F} \cap \mathfrak{G}$] means that $\langle \mathfrak{F} \rangle \vee \langle \mathfrak{G} \rangle = \mathfrak{P}(X)$ [resp., $\langle \mathfrak{F} \rangle \vee \langle \mathfrak{G} \rangle \neq \mathfrak{P}(X)$]. From convergence space theory, we have that a filter base $\mathfrak{F}$, *$\theta$-accumulates* [resp., *$\delta$-accumulates*] to $x \in X$ iff $\mathfrak{F} \cap [\mathcal{C}(x)]$ [resp., $\mathfrak{F} \cap \mathfrak{N}_s(x)$] iff for each $F \in \mathfrak{F}$ and for each $G \in [\mathcal{C}(x)]$ [resp., for each $G \in \mathfrak{N}_s(x)$], $F \cap G \neq \varnothing$. All other pertinent definitions and notation will be given in the sequel.

**3. rc-convergence.** We now define the rc-convergence structure for $X$. Let $\mathcal{C}_r(x) = \{\mathrm{cl}_X G | [x \in \mathrm{cl}_X G] \wedge [G \in T]\}$.

DEFINITION 3.1. A filter base $\mathfrak{F}$ on $X$ rc-converges to $x \in X$ if $\langle \mathcal{C}_r(x) \rangle \subset \langle \mathfrak{F} \rangle$.

It follows immediately that a filter base $\mathfrak{F}$ rc-accumulates to $x \in X$ iff $[\mathcal{C}_r(x)] \cap \mathfrak{F}$ iff for each $F \in \mathfrak{F}$ and each $R \in [\mathcal{C}_r(x)]$, $F \cap R \neq \varnothing$. In general, if a filter base rc-converges [resp., rc-accumulates], then it $\theta$-converges [resp., $\theta$-accumulates]. Let

$$\mathrm{RC}(X) = \{x | [x \subset X] \wedge [x \text{ is regular-closed}]\},$$
$$\mathrm{RO}(X) = \{x | [x \in T] \wedge [x \text{ is regular-open}]\}$$

and

$$\mathrm{SO}(X) = \{x | [x \subset X] \wedge [x \text{ is semiopen}]\}.$$

THEOREM 3.2. *For $(X, T)$, the following are equivalent*:

(i) *$X$ is $S$-closed*;

(ii) *for every cover $\mathcal{C} \subset \mathrm{RC}(X)$ there exists a finite subcover*;

(iii) *for every $\mathcal{C} \subset \mathrm{RO}(X)$ such that $\bigcap \mathcal{C} = \varnothing$, there exists a finite $\mathcal{B} \subset \mathcal{C}$ such that $\bigcap \mathcal{B} = \varnothing$*;

(iv) *every filter base on $X$ rc-accumulates*;

(v) *every maximal filter base on $X$ rc-converges.*

PROOF. (i) $\rightarrow$ (ii). This is obvious since $\mathrm{RC}(X) \subset \mathrm{SO}(X)$.

(ii) $\rightarrow$ (i). Assume $X$ is not $S$-closed. Then there exists a cover $\mathcal{C} \subset \mathrm{SO}(X)$ such that $\mathcal{C}$ has no finite proximate subcover. Thus $\{\mathrm{cl}_X V | V \in \mathcal{C}\} \subset \mathrm{RC}(X)$ has no finite subcover. The result follows from this contradiction.

(ii) $\leftrightarrow$ (iii). This is obvious since $\mathrm{RO}(X) = \{X - x | x \in \mathrm{RC}(X)\}$.

(iv) $\leftrightarrow$ (v). A maximal filter base rc-converges iff it rc-accumulates.

(v) $\leftrightarrow$ (i). We simply show that $S$-convergence in the sense of Thompson [16] is equivalent to rc-convergence. Let a filter base $\mathcal{F}$ $s$-converge to $x \in X$ and $R \in [\mathcal{C}_r(x)]$. Then $R = \mathrm{cl}_X G_1 \cap \cdots \cap \mathrm{cl}_X G_n$. Now for each $\mathrm{cl}_X G_i$, $i = 1, \ldots, n$, there exists $F_i \in \mathcal{F}$ such that $F_i \subset \mathrm{cl}_X G_i$. Hence there exists $F \in \mathcal{F}$ such that $F \subset \bigcap \{F_i\} \subset R$. On the other hand, assume that $\mathcal{F}$ rc-converges to $x \in X$ and $V \in \mathrm{SO}(X)$, $x \in V$. Then $\mathrm{cl}_X V \in \mathrm{RC}(X)$ implies that for each $V \in \mathrm{SO}(X)$ such that $x \in V$ there exists an $F \in \mathcal{F}$ such that $F \subset \mathrm{cl}_X V$.

THEOREM 3.3. *For $(X, T)$, the following are equivalent*:

(i) *$X$ is extremally disconnected*;

(ii) *if a filter base on $X$ $\delta$-converges, then it rc-converges*;

(iii) *a filter base on $X$ rc-converges iff it $\theta$-converges*;

(iv) *if a filter base on $X$ converges with respect to the topology $T$, then it rc-converges.*

PROOF. (i) $\rightarrow$ (ii). Assume that $X$ is extremally disconnected. Let $x \in \mathrm{cl}_X G$, $G \in T$. Then $\mathrm{cl}_X G \in T$. Hence $\mathrm{int}_X \mathrm{cl}_X(\mathrm{cl}_X G) = \mathrm{int}_X \mathrm{cl}_X G = \mathrm{cl}_X G$ implies that $\mathrm{cl}_X G \in \mathrm{RO}(X)$. Thus if $\mathcal{F}$ is $\delta$-convergent to $x$, then $\mathcal{F}$ is rc-convergent to $x$.

(ii) $\rightarrow$ (i). Let $G \in T$. We need to show that $\mathrm{cl}_X G \in T$. Let $x \in \mathrm{cl}_X G$. Thus $\mathcal{N}(x)$ $\delta$-converges to $x$. Hence $\mathcal{N}(x)$ rc-converges to $x$. Thus $\mathrm{cl}_X G \in T$.

(i) $\rightarrow$ (iii). Let $x \in \mathrm{cl}_X G$, $G \in T$. Since $\mathrm{cl}_X G \in T$, then $\mathrm{cl}_X G \in \mathcal{C}_r(x)$ and the result follows.

(iii) $\rightarrow$ (ii) and (iii) $\rightarrow$ (iv) are obvious.

(iv) $\rightarrow$ (i). Let $x \in \mathrm{cl}_X G$, $G \in T$. Since $\mathfrak{N}(x)$ converges to $x$, then $\mathfrak{N}(x)$ rc-converges to $x$. Hence there exists some $H \in T$ such that $x \in H \subset \mathrm{cl}_X G$. Thus $\mathrm{cl}_X G \in T$.

It is known [**1**, p. 159] that for Hausdorff spaces $X$ is extremally disconnected iff $X_s$ is extremally disconnected. This result holds for non-Hausdorff spaces as well. We now improve somewhat upon the results in [**16**].

THEOREM 3.4. *If $X$ is nearly-compact (or quasi-$H$-closed) and extremally disconnected, then $X$ is $S$-closed.*

PROOF. Assume $X$ is nearly-compact. Let $\mathfrak{U}$ be a maximal filter base on $X$. Then $\mathfrak{U}$ $\delta$-converges to some $x \in X$. Theorem 3.3 implies that $\mathfrak{U}$ rc-converges. The proof for quasi-$H$-closed is similar.

THEOREM 3.5. *If $X$ is almost-regular and $S$-closed, then $X$ is extremally disconnected and nearly-compact.*

PROOF. Assume $X$ is not extremally disconnected. Then there exists $G \in \mathrm{RO}(X)$ such that $\mathrm{cl}_X G - G \neq \varnothing$ and $X - \mathrm{cl}_X G \neq \varnothing$. Let $x \in \mathrm{cl}_X G - G$. Now $\mathfrak{N}(x) \cap [\{G\}]$ and $\mathrm{cl}_X G$ is $S$-closed relative to $X$. Thus there exists $p \in \mathrm{cl}_X G$ such that $\mathfrak{N}(x) \cap [\{G\}] \cap [\mathcal{C}_r(p)]$. If $p \in \mathrm{cl}_X G - G$, then $p \in X - G$ implies that $\mathfrak{N}(x) \cap [\{G\}] \cap [\{X - G\}]$ since $X - G \in \mathrm{RC}(X)$ and $X - G \in \langle \mathcal{C}_r(p) \rangle$. This contradiction yields that $G \in \mathfrak{N}(p)$. Almost-regularity and $G \in \mathrm{RO}(X)$ imply that $G \in \langle \mathcal{C}(p) \rangle$. Since $x \in G$, then $\mathcal{C}(p) \perp \mathfrak{N}(x)$. This contradicts the fact that $\mathfrak{N}(x)$ rc-accumulates to $p$. Hence $X$ is extremally disconnected. Let $\mathfrak{U}$ be a maximal filter base. Then $S$-closedness implies that $\mathfrak{U}$ rc-converges to some $x \in X$. Theorem 3.3(iii) implies $\mathfrak{U}$ $\theta$-converges to $x$. Almost-regularity implies that $\mathfrak{U}$ $\delta$-converges to $x$. Since $X$ is nearly-compact iff every maximal filter base $\delta$-converges, then this completes the proof.

COROLLARY 3.6. *Let $X$ be almost-regular. Then $X$ is $S$-closed iff $X$ is nearly-compact and extremally disconnected iff $X_s$ is regular, compact and extremally disconnected.*

THEOREM 3.7. *If $X$ is weakly-$T_2$ and $S$-closed, then $X$ is $H$-closed and extremally disconnected.*

PROOF. Assume $X$ is weakly-$T_2$ and distinct $x, y \in X$. Hence there exists $G' \in \mathrm{RO}(X)$ and $F$ such that $X - F \in \mathrm{RO}(X)$, $x \in G'$, $y \in F$ and $G' \cap F = \varnothing$. Thus $\mathfrak{N}_s(x) \perp [\mathcal{C}_r(y)]$. Assume $X$ is not extremally disconnected. Then there exists $G \in \mathrm{RO}(X)$ such that $\mathrm{cl}_X G - G \neq \varnothing$ and $X - \mathrm{cl}_X G \neq \varnothing$. Let $x \in \mathrm{cl}_X G - G$. Then $\mathfrak{N}_s(x) \cap [\{G\}]$. Since $\mathrm{cl}_X G$ is $S$-closed relative to $X$, there exists $y \in \mathrm{cl}_X G$ such that $\mathfrak{N}_s(x) \cap [\{G\}] \cap [\mathcal{C}_r(y)]$. Weakly-$T_2$ implies $x = y$. As in the proof of Theorem 3.4 we have that $G \in_s \mathfrak{N}(x)$. This contradiction implies that $X$ is extremally disconnected. Theorem 3.3(iii) and the fact that a space is Hausdorff iff for distinct $x, y$, $\mathfrak{N}_s(x) \perp \mathcal{C}(y)$ and a space is quasi-$H$-closed iff each maximal filter base $\theta$-converges yield that $X$ is $H$-closed.

COROLLARY 3.8. *If $X$ is weakly-$T_2$ and extremally disconnected, then $X$ is Hausdorff.*

COROLLARY 3.9. *If $X$ is weakly-$T_2$ nearly-compact (or quasi-$H$-closed) and extremally disconnected, then $X$ is $S$-closed and Hausdorff.*

COROLLARY 3.10. *Let $X$ be weakly-$T_2$. Then $X$ is $S$-closed iff $X$ is quasi-$H$-closed and extremally disconnected iff $X_s$ is compact Hausdorff and extremally disconnected.*

COROLLARY 3.11. *In the category of nearly-compact Hausdorff spaces and $\theta$-continuous maps, the $S$-closed spaces are precisely the projective objects.*

REMARK 3.12. A map $f$: $(X, T) \to (Y, \tau)$ is $\theta$-continuous if for each $G \in \tau$ such that $f(x) \in G$ there exist $V \in T$, $x \in V$ and $f(\mathrm{cl}_X V) \subset \mathrm{cl}_Y G$.

Using the semiregularization, it follows that Theorem 3 of [16] and the corollary to Theorem 4 of [16] hold for almost-regular spaces. Thompson explicitly gives only compact examples of $S$-closed spaces, in particular $\beta(N)$. Let $\mathcal{I}(X)$ denote the set of all isolated elements of $X$ and $\sigma(X)$ the Fomin extension [3].

THEOREM 3.13. *Let $X$ be noncompact, weakly-$T_2$ and extremally disconnected. If $\sigma(X)$ is compact, then $\sigma(X) = \beta(X)$ and $X - \mathcal{I}(X)$ is compact.*

PROOF. By Theorem 6.2 of [11] there exists a continuous surjection from $\sigma(X)$ onto $\beta(X)$. Thus $\sigma(X) = \beta(X)$. Lemma 5 of [8] implies that $X - \mathcal{I}(X)$ is compact.

COROLLARY 3.14. *Let $X$ be noncompact, weakly-$T_2$ and extremally disconnected. If $\sigma(X)$ is compact, then $\sigma(X) = \beta(X)$ and $\mathcal{I}(X)$ is infinite.*

COROLLARY 3.15. *Let $X$ be any discrete space. Then $\beta(X)$ is $S$-closed.*

THEOREM 3.16. *If $X$ is weakly-$T_2$ and extremally disconnected, then $X$ is a dense subspace of the $S$-closed Hausdorff, almost-regular space $\sigma(X)$.*

EXAMPLE 3.17. Let $(X, T)$ be infinite and discrete. Let $\mathcal{U}$ be the set of all nonprincipal ultrafilters on $X$. Then $Y = X \cup \mathcal{U}$ with the topology generated by $T$ and all sets of the form $F \cup \{\mathcal{F}\}$, $F \in \mathcal{F} \in \mathcal{U}$, is known to be Hausdorff and extremally disconnected. Since $Y - \mathcal{I}(Y) = \mathcal{U}$ and $\mathcal{U}$ is noncompact, then by Theorem 3.13, $\sigma(Y)$ is noncompact, Hausdorff and $S$-closed.

**4. Mapping theory.** It is known that $S$-closedness is not preserved by continuous maps [15]. A map $f$: $(X, T) \to (Y, \tau)$ is *almost-open* [4] [resp., *W-almost-open* [9]] if for each $G \in \mathrm{RO}(X)$, $f[G] \in \tau$ [resp., for each $V \in \tau$, $f^{-1}[\mathrm{cl}_Y V] \subset \mathrm{cl}_X(f^{-1}[V])$]. Almost-open in [18] implies $W$-almost-open. A map is *almost-continuous* [10] [resp., *w$\theta$-continuous* [10]] iff for each $x \in X$ and open neighborhood $V$ of $f(x)$ there exists an open neighborhood $G$ of $x$ such that $f[G] \subset \mathrm{int}_Y \mathrm{cl}_Y V$ [resp., $f[G] \subset \mathrm{cl}_Y V$].

THEOREM 4.1. *A W-almost-open and w$\theta$-continuous map $f$: $(X, T) \to (Y, \tau)$ is almost-continuous.*

PROOF. It is known that for $V \in \tau$, $\mathrm{cl}_X(f^{-1}[V]) \subset f^{-1}[\mathrm{cl}_Y V]$ [10]. $W$-almost-open implies $\mathrm{cl}_X(f^{-1}[V]) = f^{-1}[\mathrm{cl}_Y V]$. This global condition implies that $f$ is almost-continuous.

THEOREM 4.2. *An almost-open almost-continuous map $f$: $(X, T) \to (Y, \tau)$ preserves rc-convergence.*

PROOF. Let $F \in \mathrm{RC}(Y)$. Then $f^{-1}[F] \in \mathrm{RC}(X)$. Thus if $f(x) \in \mathrm{cl}_Y V$ for arbitrary $V \in \tau$, then $f^{-1}[\mathrm{cl}_Y V] = \mathrm{cl}_X H$, $H \in T$. Consequently, $f^{-1}[\mathrm{cl}_Y V] \in \mathcal{C}_r(x)$. Hence $\langle \mathcal{C}_r(f(x)) \rangle \subset f(\langle \mathcal{C}_r(x) \rangle)$ and the result follows.

THEOREM 4.3. *A W-almost-open and w$\theta$-continuous map $f$: $(X, T) \to (Y, \tau)$ preserves rc-convergence.*

PROOF. Let $f(x) \in \mathrm{cl}_Y V$, $V \in \tau$. Then $x \in f^{-1}[\mathrm{cl}_Y V]$ and $f^{-1}[\mathrm{int}_Y \mathrm{cl}_Y V] = G \in T$. $W$-almost-open implies that

$$\mathrm{cl}_X\left(f^{-1}[V]\right) \subset \mathrm{cl}_X G \subset \mathrm{cl}_X\left(f^{-1}[\mathrm{cl}_Y V]\right) \subset \mathrm{cl}_X\left(f^{-1}[V]\right)$$

which implies that $\mathrm{cl}_X G = \mathrm{cl}_X(f^{-1}[\mathrm{cl}_Y V])$. Almost continuity yields that $x \in \mathrm{cl}_X G \subset f^{-1}[\mathrm{cl}_Y V]$. Hence $f^{-1}[\langle \mathcal{C}_r(f(x)) \rangle] \subset \langle \mathcal{C}_r(x) \rangle$.

As a final result we locate $\sigma(X)$ within the class of all $S$-closed extensions of a space $X$.

THEOREM 4.4. *Let $X$ be weakly-$T_2$, extremally disconnected and $\mathfrak{S}$ the class of all weakly-$T_2$ S-closed extensions of $X$. Then for each $Z \in \mathfrak{S}$ there exists an almost-continuous surjection $F$: $\sigma(X) \to Z$ such that $F|X$ = identity (i.e., $\sigma(X)$ is the almost-projective maximum in $\mathfrak{S}$).*

PROOF. In [7], it is shown that for each $Z \in \mathfrak{S}$ there exists a $\theta$-continuous surjection $F$: $\sigma(X) \to Z$ such that $F|X$ = identity. Since $Z$ is almost-regular, then $F$ is almost-continuous.

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