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# DERIVATIONS ON CONTINUOUS FUNCTIONS

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ABSTRACT. We shall give a simple proof of Sakai's characterization on derivations of continuous functions.

A derivation $d$ of an algebra $A$ is a linear map of the definition domain $D$ of $d$ into $A$ such that $d(fg) = (df)g + f(dg)$ for $f, g \in D$. S. Sakai [1] discussed derivations of $C[0, 1]$ and gave several theorems to represent by concrete operations. We shall prove a theorem of Sakai's type with a different simple proof:

THEOREM. *If the definition domain of a derivation $d$ of $C[0, 1]$ includes the space $C^{(\infty)}[0, 1]$ of all infinitely differentiable functions, then there is (unique) $h \in C[0, 1]$ such that $df = hf'$ for all $f \in C^{(\infty)}[0, 1]$.*

PROOF. For every $f \in C^{(\infty)}[0, 1]$ and $0 < a < 1$ there is a certain $g \in C^{(\infty)}[0, 1]$ such that

$$f(x) = f(a) + f'(a)(x - a) + g(x)(x - a)^2$$

so that we have

$$df(x) = f'(a)dx + dg(x)(x - a)^2 + 2g(x)(x - a)d(x - a).$$

Putting $h = dx$ and $x = a$, we have $df(a) = f'(a)h(a)$.

## REFERENCES

1. S. Sakai, *Recent developments in the theory of unbounded derivations in $C^*$-algebras*, $C^*$-Algebras and Applications to Physics, Lecture Notes in Math., vol. 650, Springer-Verlag, Berlin and New York, 1978, pp. 85–122.

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