THE STABILITY OF DE RHAM CURRENTS

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Abstract. The purpose of this short note is to give a theorem concerning the action of the diffeomorphism group of a smooth manifold on the space of de Rham currents.

Let $M$ be a paracompact, orientable $n$-dimensional, $C^\infty$-manifold. We shall denote by $\mathcal{D}^p(M)$ [resp. $\mathcal{L}^p(M)$] the space of de Rham currents [resp. closed de Rham currents] on $M$, endowed with the uniform convergence topology [resp. induced topology] [3].

It is easy to see that the notions of local and global stability for differential forms [1], [2] can be extended, in natural way, for de Rham currents.

The following theorem shows that there exist no globally defined stable de Rham currents on $M$.

Theorem. Let $T \in \mathcal{D}^p(M)$. Then for any neighborhood $V_T$ of $T$ there exists a $p$-current $S$ in $V_T$ such that for any smooth diffeomorphism $f: M \rightarrow M$ one has $f(S) \neq T$.

The sketch of the proof. We shall suppose that the theorem is not true. First one considers the case when $T$ is a form-like de Rham current (i.e., a current induced by a form), and then the case when $T$ is not a form-like de Rham current. Using the density of Dirac currents for the first case and the density of form-like currents for the second case, one obtains immediately a contradiction. Q.E.D.

So the notion of global stability for de Rham currents does not make sense.

Remark. Assuming that $M$ is a compact manifold without boundary, and taking into account the Hodge-Kodaira-de Rham decomposition theorem [3], the above theorem is also true if one replaces $\mathcal{D}^p(M)$ with $\mathcal{L}^p(M)$.

Bibliography


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