PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 1, September 1981

ON THE EQUIVARIANT HOMOTOPY TYPE OF G-ANR'S

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ABSTRACT. I show that every metric G-ANR has the G-homotopy type of a G-CW complex. Therefore I. James and G. Segal's results concerning equivariant homotopy type are special cases of the Whitehead theorem for G-CW complexes.

In this note G is assumed to be a compact Lie group. A metric G-space X is said to be a G-ANR if for any G-embedding $i: X \to Y$ in a metric G-space Y such that iX is closed in Y, the image iX is a G-retract of some open invariant neighborhood of Y.

THEOREM. *Every metric G-ANR has the G-homotopy type of a G-CW complex.*

As a consequence of this theorem and Theorem 5.3 in [3] we obtain the following.

COROLLARY (JAMES, SEGAL [1]). *Let $f: X \to Y$ be a G map between G-ANR's. Then f is a G-homotopy equivalence iff $f^H: X^H \to Y^H$ is an ordinary homotopy equivalence for every closed subgroup $H \subseteq G$.*

PROOF OF THEOREM. Let X be a metric G-ANR. By the equivariant version of a standard argument (Lemma 4.7 of [4]), it suffices to prove that X is G-dominated by a G-CW complex. Observe that every metric G-space X may be G-embedded as a closed G-subset of a convex G-set in a Banach space of bounded real-valued functions on X with G-action given by $g(h)(x) = h(g^{-1}(x))$ for $g \in G$, $h: X \to R$, and $x \in X$.

Therefore we may assume that X is a closed G-subset of a convex G-set C in a Banach G-space.

Being a G-ANR, X is a G-retract of some open neighborhood U of X in C; in particular it is G-dominated by some G-CW complex. This is seen by an easy modification of the proof of Theorem 3.B in [2]. We obtain that U is G-dominated by a G-nerve induced by a locally finite refined slice covering and this G-nerve has the G-homotopy type of a G-CW complex (which is a direct limit of barycentric manifolds in the notation of [2]).

REMARK. Matumoto's proof that a barycentric manifold is a G-CW complex is incorrect because it relies on a result of Yang which relies on an incorrect result of Cairns. A correct proof that smooth manifolds are G-CW complexes is in a preprint *Triangulation of stratified fibre bundles* by Andrei Verona. The weaker

Received by the editors August 8, 1980.
AMS (MOS) subject classifications (1970). Primary 57E10.

0002-9939/81/0000-0443/$01.50

assertion that smooth G-manifolds have the G-homotopy type of G-CW complexes (which is all that might be needed for this paper is immediate from [**4**]).

I wish to thank the referee for his helpful suggestions.

References

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